Efficiency, Welfare, and Political Competition*

Felix J. Bierbrauer† and Pierre C. Boyer‡

June 24, 2015

Abstract

We study political competition in an environment in which voters have private information about their preferences. Our framework covers models of income taxation, public-goods provision, or publicly provided private goods. Politicians are vote-share-maximizers. They can propose any policy that is resource-feasible and incentive-compatible. They can also offer special favors to subsets of the electorate. We prove two main results. First, the unique symmetric equilibrium is such that policies are surplus-maximizing and hence first-best Pareto-efficient. Second, there is a surplus-maximizing policy that wins a majority against any welfare-maximizing policy. Thus, in our model, policies that trade off equity and efficiency considerations are politically infeasible.

Keywords: Political Competition; Asymmetric Information; Public Goods; Nonlinear Income Taxation; Redistributive Politics.

JEL classification: C72; D72; D82; H21; H41; H42.

*We are grateful to the editors Elhanan Helpman and Robert Barro, and to three anonymous referees for their comments and suggestions. We also thank Sören Blomquist, Philippe De Donder, Hans Peter Grüner, Emanuel Hansen, Bas Jacobs, Laurence Jacquet, Eckhard Janeba, Philippe Jehiel, Hideo Konishi, Kai Konrad, Yukio Koriyama, Jean-François Laslier, Alessandro Lizzeri, Michel Le Breton, Étienne Lehmann, Thomas Mariotti, Benny Moldovanu, Florian Morath, Brian Roberson, Dominik Sachs, Jenny Simon, Laurent Simula, Thomas Tröger, Carl Christian von Weizsäcker, and Nicolas Werquin for very helpful discussions, as well as seminar and conference participants at Princeton University, École Polytechnique, Boston College, Toulouse School of Economics, Marburg University, SITE Stockolm, Uppsala University, SFB-TR15 in Berlin, Paris School of Economics, MPI Bonn, PET 2013 in Lisbon, ENS-Cachan, Workshop on Voting Theory and Elections at Cergy-Pontoise, Workshop on the Economics of Taxation at ENS-Cachan, IX Tax-Day at MPI in Munich, 2014 ASSA meeting in Philadelphia, 2014 Tax Theory Conference in Cologne, 2014 Workshop on Political Economy in Paris, CESifo Area Conference on Public Sector Economics 2015, and ZEW Public Finance Conference 2015. The second author gratefully acknowledges the Collaborative Research Center 884 for financial support. The second author also thanks Thema at University Cergy-Pointoise and ENS-Cachan for their hospitality and financial support. The usual disclaimer applies.

†CMR - Center for Macroeconomic Research, University of Cologne, Albert-Magnus Platz, 50923 Köln, Germany. E-mail: bierbrauer@wiso.uni-koeln.de

‡University of Mannheim, Department of Economics, 68131 Mannheim, Germany. E-mail: pierre.boyer@uni-mannheim.de

1 Introduction

Mechanism design has become the dominant paradigm for a normative analysis of publicly provided goods or tax systems. The strength of this approach is that it provides a rigorous justification of the constraints that a policy-maker faces. Available technologies and endowments give rise to resource constraints, privately held information gives rise to incentive compatibility constraints, predetermined institutional arrangements may generate another layer of constraints such as, for instance, the requirement of voluntary participation. Political economy approaches, by contrast, often impose additional restrictions on the set of admissible policies. These restrictions lack a theoretical foundation, but are imposed for pragmatic reasons, e.g. because they ensure the existence of a Condorcet winner in a model of political competition.

For instance, for redistributive income taxation, a normative analysis in the tradition of Mirrlees (1971) characterizes a welfare-maximizing income tax with no *a priori* assumption on the functional form of the tax function. A well-known political economy approach to this problem by Meltzer and Richard (1981) is based on the assumption that all individuals face the same marginal tax rate, and that tax revenues are used to finance a uniform lump-sum transfer to all citizens. One can thereby show that the preferred policy of the voter with median income wins a majority against any alternative policy proposal. This result, however, does not extend to the domain of non-linear income tax schedules: The median voter's preferred non-linear income tax schedule does not win a majority against an alternative tax schedule under which the median level of income is taxed more heavily but all other incomes are treated more favorably.

More generally, the use of different models in normative and positive public finance makes it difficult to provide answers to the following questions: Does political competition generate Pareto-efficient outcomes? Does it generate welfare-maximizing outcomes? Is there a sense in which political competition gives rise to political failures, in analogy to the theory of market failures.

In this paper, we study political competition from a mechanism design perspective. Our framework covers both publicly provided goods and redistributive income taxation. We ask which mechanism emerges as a result of political competition under the assumption that **a** politician's objective is to win an election, as in Downs (1957). Politicians can propose any mechanism that is incentive-compatible and resource-feasible. Moreover, we consider a policy domain that is larger than the one usually considered in normative treatments of public goods provision or income taxation: We give politicians the possibility to accompany, say, a proposed income tax schedule with a distribution of favors in the electorate. These favors are unrelated to the voters's preferences for publicly provided goods or their productive abilities, and, they would not be needed to achieve a Pareto-efficient, or welfare-maximizing outcome. Politicians may still want to use them

so as to generate more support for their policy platform. In political economy analysis, this is often referred to as pork-barrel spending.[1] Finally, we assume that the preferences of voters are quasi-linear in the consumption of private goods.[2]

We prove two theorems. Theorem 1 can be interpreted as a first welfare theorem for political competition. It claims that, in any symmetric equilibrium, both politicians propose a surplus-maximizing policy. Thus, the political equilibrium allocation cannot be Pareto-improved upon in the set of resource-feasible policies. For a problem of income taxation, Theorem 1 implies that, in a political equilibrium, there is no use of distortionary tax instruments. Transfers of resources between voters take place in equilibrium, but are financed exclusively with non-distortionary lump-sum transfers. For a model that involves the provision of a non-rival good, such as clean air or national defense, Theorem 1 implies a political equilibrium allocation that satisfies the efficiency condition which is known as the Samuelson rule, Samuelson (1954). For a model with publicly provided private goods, such as health care or education, Theorem 1 implies that the political equilibrium allocation gives rise to the same consumption levels as a competitive market allocation, i.e. marginal benefits of consumption are equalized across voters.

Theorem 2 is concerned with the question whether welfare-maximizing policies have a chance in the political process. To formalize this question, we introduce an assumption of risk-aversion. This assumption implies that a welfare-maximizing policy does not involve pork-barrel spending: From an ex-ante perspective, all voters prefer an equal treatment over a random allocation of special treatments. We impose no further restriction on the set of welfare-maximizing policies, i.e. any policy that maximizes a weighted average of the voters' utility levels over the set of incentive-compatible and feasible policies will be referred to as a welfare-maximizing policy. Theorem 2 then asserts that there is a policy that wins a majority against any welfare-maximizing policy. This policy is surplus-maximizing and involves a random allocation of special treatments. For a model of income taxation, the theorem implies that any welfare-maximizing policy that involves a transfer of resources from richer individuals to poorer individuals can be defeated by a policy that has no such transfers, but distributes favors in such a way that many rich and some poor voters will be attracted.

To illustrate these insights, suppose that there are only two groups of individuals.[3] A large group of individuals with low productive abilities and a small group of more productive individuals. The more productive individuals have a comparatively low cost of productive effort, so that when confronted with an income tax schedule, these individuals choose a high level of effort and therefore end up being richer than the less productive

[1] Myerson (1993) and Lizzeri and Persico (2001) characterize equilibrium pork-barrel spending for economies with complete information on preferences and technologies. We use some of their insights. Our focus, however, is on environments with private information.

[2] **In the Online-Appendix for this paper, we allow for more general preferences and, in particular, for income effects.**

[3] This two-type example is explicitly worked out in the Online-Appendix.

individuals. If pork-barrel spending is excluded from the analysis, a political equilibrium gives rise to the preferred income tax schedule of the larger group, here the income tax schedule that maximizes a Rawlsian welfare function. Theorem 2 implies that a surplus-maximizing policy combined with a particular distribution of favors in the electorate will defeat this policy. This surplus-maximizing policy will be supported by a certain fraction of the poor, namely those who receive a lot of pork, and an even larger fraction of the rich, who benefit also from getting rid of the Rawlsian income tax schedule. The total effect is that a majority votes against the Rawlsian policy.

Our analysis of the income tax problem generates the prediction that marginal tax rates are zero at all levels of income. This is at odds with reality: Tax systems in developed countries include distortionary income taxes. Therefore, our analysis is interesting only as a theoretical benchmark. It provokes the question what additional assumptions would be needed to generate positive marginal tax rates in a political equilibrium. A key feature of our analysis is that there are no party loyalties. Each voter simply supports the party that offers the better outcome. In an extension of our model we drop this assumption and suppose instead that voters trade-off idiosyncratic party preferences against the payoffs they realize under the parties' policy proposals. We do not restrict the distribution of these idiosyncratic party preferences in the electorate, e.g. we allow for the possibility that, say, rich voters are more likely to vote for party 1 and that middle-class voters are more likely to vote for party 2. We show that this framework is capable of generating positive marginal tax rates in equilibrium. To this end we derive an analogue to the well-known ABC-formula due to Diamond (1998) that characterizes welfare-maximizing tax rates. In our formula, welfare weights are replaced by political weights, i.e. by a measure of how many voters of a particular income category a party can attract by lowering their marginal tax rate. Thus, the less responsive voters are to changes in tax rates, the higher will be their marginal tax rate in a political equilibrium.

The remainder is organized as follows. The next section reviews the related literature. Section 3 contains our main results for a model of publicly provided goods. Section 4 explains the logic of the argument and develops an intuition for the main results. Section 5 extends the analysis to a model of income taxation. We discuss equilibrium existence in Section 6. Section 7 contains the analysis of a probabilistic voting model. Section 8 clarifies how alternative modeling choices would affect our results. **In particular, we discuss to what extent our results extend if we depart from quasi-linear preferences and allow for income effects.** The last section contains concluding remarks. All proofs are relegated to the Appendix. In the body of the text, we present our results under the assumptions that the set of possible voter types is a compact interval and that there is a continuum of voters. The analysis extends both to a model with a discrete type space and to a model with a finite number of voters. These extensions are dealt with in the Online-Appendix for this paper.

2 Related literature

Our basic setup is taken from the normative literature that uses a mechanism design approach to study public goods provision, or income taxation. The main difference between our approach and this literature is that we replace the fictitious benevolent mechanism designer by the forces of political competition. The normative literature on public goods provision has, by and large, focussed on the question whether surplus-maximizing outcomes can be obtained if incentive compatibility and/or participation constraints have to be respected.[4] We do not impose participation constraints in our analysis. We take it as given that the government uses its coercive power to finance publicly provided goods. The literature on optimal income taxation in the tradition of Mirrlees (1971) has analyzed the conditions under which a welfare-maximizing policy involves distortionary taxes.[5]

Political economy approaches to redistributive income taxation or public goods provision have led to different formulations of median voter theorems. Roberts (1977) and Meltzer and Richard (1981) study political competition using the model of linear income taxation due to Sheshinski (1972), i.e. it is assumed that there is linear tax on income and that the revenues are used in order to finance a uniform lump-sum transfer. With this policy domain, preferences are single-peaked so that the median voter's preferred policy emerges as the outcome of competition between two vote-share maximizing parties. This literature is known for the prediction, due to Meltzer and Richard (1981), that the equilibrium tax rate is an increasing function of the gap between the median voter's income and the average income in the economy. Röell (2012) and Brett and Weymark (2014), by contrast, study political competition over non-linear income taxes. However, they do not consider competition between vote-share-maximizing politicians. Instead, they study competition between citizen-candidates, see Osborne and Slivinski (1996) and Besley and Coate (1997). This enables them to prove a median-voter-theorem: The equilibrium tax policy is the non-linear income tax schedule that is preferred by the voter with the median level of income. In all these papers, politicians are not given the opportunity to engage in pork-barrel spending. As follows from our analysis, a removal of these restrictions yields different results. The equilibrium policy that we characterize in Corollaries 1 and 2 wins a majority against the median voter's preferred policy.

There also is a literature on the political economy of taxation and public spending in dynamic models. A variety of political economy models has been explored by this literature.[6] However, to the best of our knowledge, there is not yet an analysis of Downsian

[4]Important references are Green and Laffont (1977), d'Aspremont and Gérard-Varet (1979), Güth and Hellwig (1986), and Mailath and Postlewaite (1990).

[5]Our framework covers this model under the assumption that preferences are quasi-linear in private-goods consumption. This is a special case which has received considerable attention in the literature on optimal taxation, see e.g. Diamond (1998) and Saez (2001).

[6]Besley and Coate (1998) use the citizen-candidate-model, Acemoglu, Golosov and Tsyvinski (2008;

competition.

Our work is related to the game-theoretic literature on the "divide-the-dollar-game". These are models of political competition in which a policy proposal specifies how a cake of a given size should be distributed among voters. Our model differs in that policy proposals affect the size of the cake that is available for redistribution. Also, there is private information on preferences so that not only resource constraints but also incentive compatibility constraints have to be taken into account. Still our equilibrium characterization makes use of insights which have been provided by this literature, in particular by Myerson (1993).[7] Lizzeri and Persico (2001; 2004; 2005), Roberson (2008), and Crutzen and Sahuguet (2009) extend this framework so as to characterize political equilibria under the assumption that politicians face a choice between a policy with wide-spread social benefits and a policy that is targeted to specific voters. They characterize the conditions under which political failures occur in the sense that political equilibrium allocations are not efficient.

The literatures on pork-barrel-spending and the divide-the-dollar-game are related in that both study the allocation of public funds across different economic entities. A frequent example is that politicians channel tax revenue to different regions where the money is then used to finance the provision of local public goods (see, e.g., Drazen, 2000; Persson and Tabellini, 2000; Roberson, 2008; Grossman and Helpman, 2008). In empirical studies, pork-barrel spending is often used as a synonym for public spending that is targeted to a narrow set of beneficiaries – such as an infrastructure investment in a particular region, – and distinguished from spending that has wide-spread benefits, such as resources that are devoted to the judicial system or welfare programs for the population at large (see, e.g., Gagliarducci, Nannicini and Naticchioni, 2011; Funk and Gathmann, 2013).

Various papers study non-linear income taxation in models with multiple jurisdictions and mobile workers, see Wilson (1980; 1992), Bierbrauer, Brett and Weymark (2013), or Lehmann, Simula and Trannoy (2014). This literature finds that income tax competition tends to reduce marginal tax rates relative to the Mirrleesian analysis of an economy with immobile workers. Our paper is related in that we also find that competition reduces distortions. The underlying mechanism, however, is very different. We stick to the Mirrleesian benchmark model in that we assume that workers are immobile. The analysis of tax competition by Lehmann, Simula and Trannoy (2014) employs the random participation model due to Rochet and Stole (2002), a framework that has also been used to study

2010) employ the political agency model by Barro (1973) and Ferejohn (1986), Battaglini and Coate (2008) invoke the legislative bargaining model due to Baron and Ferejohn (1989) and Fahri and Werning (2008) use of the probabilistic voting due to Lindbeck and Weibull (1987). Finally, Martimort (2001) studies partisan politics.

[7] Other contributions to this literature include Lizzeri (1999), Laslier and Picard (2002), Roberson (2006), Sahuguet and Persico (2006), Carbonell-Nicolau and Ok (2007), Kovenock and Roberson (2009), Casamatta, Cremer and De Donder (2010), and Roemer (2011).

competition between profit-maximizing firms that offer non-linear price schedules.[8] For instance, Armstrong and Vickers (2001) find that this gives rise to first-best outcomes if individuals differ not only in the willingness to pay for the goods offered by these firms but also by their transportation costs to the firms' locations. Our analysis of the probabilistic voting model in Section 7 uses a similar framework to study the conditions under which a political equilibrium gives rise to distortionary taxation.

3 Publicly provided goods

We begin with a framework in which a policy specifies the public provision and financing of a public or private good. In Section 5 we extend our analysis to a model of income taxation.

3.1 The economic environment

Preferences. There is a continuum of voters of measure 1. The set of voters is denoted by I, with typical element i. We denote by q_i voter i's consumption of the publicly provided good. Individual i's valuation of the good is given by a function $v(\theta_i, q_i)$, where θ_i is referred to as person i's type. The set of types is denoted by $\Theta \subset \mathbb{R}_+$. We assume that Θ is a compact interval, so that $\Theta = [\underline{\theta}, \overline{\theta}]$.

The function v is assumed to have the following properties: Zero consumption gives zero utility, for all $\theta_i \in \Theta$, $v(\theta_i, 0) = 0$. The lowest type does not benefit from public goods provision, for all q_i, $v(\underline{\theta}, q_i) = 0$. For all other types, the marginal benefit from increased consumption is positive and decreasing so that for all $\theta_i > \underline{\theta}$ and all q_i, $v_2(\theta_i, q_i) > 0$ and $v_{22}(\theta_i, q_i) \leq 0$. The marginal benefit of consumption is increasing in the individual's type, for all $\theta \in (\underline{\theta}, \bar{\theta})$ and all $q_i > 0$, $v_{12}(\theta_i, q_i) > 0$.

Each individual privately observes his type. From an outsider's perspective, types of different voters are drawn independently and they are identically distributed. We denote by f the probability mass or density function and by F the cumulative distribution function. When we use an expectation operator in the following, then expectations are taken with respect to this distribution. We appeal to a law of large numbers for large economies,[9] so that we can also interpret F as the empirical cross-section distribution of types.

Policies. A policy p consists of a provision rule which determines an individual's consumption of the publicly provided good as a function of the individual's type, a rule that determines how an individual's private goods consumption depends on his type, and a

[8] These papers belong to a larger literature on competing mechanisms, see Martimort (2006) for a survey.

[9] See Judd (1985), Feldman and Gilles (1985), Al-Najjar (2004), or Sun (2006).

distribution of favors in the electorate (pork-barrel spending). The latter enables policy-makers to make specific promises to subsets of the electorate. Formally, a policy is a triple $p = (q, c, G)$, where $q : \Theta \to \mathbb{R}_+$ is the provision rule for the publicly provided good, $c : \Theta \to \mathbb{R}$ determines private goods consumption and $G : \mathbb{R}_+ \to [0, 1]$ is a *cdf* which characterizes the distribution of favors. Specifically, we follow Myerson (1993) and assume that the favors offered to different voters are *iid* random variables with cumulative distribution function G. We appeal once more to the law of large numbers for large economies and interpret $G(x)$ not only as the probability that any one individual receives an offer weakly smaller than x, but also as the population share of voters who receive such an offer.

Importantly, the distribution of favors is orthogonal to the distribution of types.[10] Thus, favors need not literally be thought of as being specific to individuals. For instance, if we take I to be a set of districts, then "pork for i" can be interpreted as money targeted to the provision of local public goods in region i. Our analysis applies to this setup under the assumption that I consists of districts of equal size and with an identical distribution of preferences. The distribution G then specifies a cross-section distribution of transfers across districts.

If individual i receives a draw x from the distribution G, then the individual's private goods consumption equals $x + c(\theta)$ if $\theta_i = \theta$. Hence, the draw x from G is a shifting parameter for the individual's consumption schedule c. Incentive compatibility, formally introduced below, will imply that individuals with higher types consume (weakly) more of the publicly provided good and therefore have to accept lower private goods consumption. Consequently, c will be a non-increasing function. We impose the normalization that $c(\overline{\theta}) = 0$. Combined with the assumption that the support of the distribution G is bounded from below by 0, this implies that non-negativity constraints on private goods consumption levels can be safely ignored in the following.[11]

Given a policy $p = (q, c, G)$, we denote the utility level that is realized by an individual with type θ and a draw x from the lottery by

$$x + u(\theta \mid q, c) = x + c(\theta) + v(\theta, q(\theta)) \ .$$

Admissible policies. Individuals have private information on their types, which implies that a policy has to satisfy the following incentive compatibility constraints: For all θ and θ',

$$v(\theta, q(\theta)) + c(\theta) \geq v(\theta, q(\theta')) + c(\theta') \ .$$

[10] We comment on the type-dependent favors in Section 8. There we argue that if such transfers were allowed for politicians would refrain from using them in equilibrium.

[11] We could adopt a different normalization. If we chose $c(\overline{\theta}) = \bar{x}$, for some $\bar{x} \neq 0$, then the equilibrium distribution of favors would have a support with minimal element equal to $-\bar{x}$.

In addition, policies have to be feasible. Let e be the economy's initial endowment with the private consumption good. We assume that e is a large number. Feasibility holds provided that

$$\int_0^\infty x \, dG(x) + E[c(\theta)] + K(q) \leq e \; . \tag{1}$$

In addition, feasibility requires that, for each θ, $q(\theta)$ belongs to a consumption set $\Lambda(q)$. We consider consumption sets that depend on the provision rule q. As explained below, this allows to cover both publicly provided private goods and pure public goods. We assume, for simplicity, that $\Lambda(q)$ is bounded from above, for all provision rules q.[12] The function K gives the resource requirement of using provision rule q. We assume that the function K is non-decreasing in $q(\theta)$, for all θ, i.e. if the consumption of individuals of type θ goes up, while all other consumption levels remain constant, then the provision costs $K(q)$ cannot fall. This framework covers the following setups:

Pure public goods: With a good that is non-rival only $\bar{q} := \max_{\theta \in \Theta} \; q(\theta)$ matters for the cost $K(q)$. We may therefore take $K(q)$ to be equal to $k(\bar{q})$, where k is an increasing and convex cost function. If the good is non-excludable, then the consumption set $\Lambda(q)$ is such that $q(\theta) \in \Lambda(q)$ if and only if $q(\theta) = \bar{q}$, for all θ.

Excludable public goods: An excludable public good has the same cost structure as a pure public good, but the consumption set changes due to excludability: $q(\theta) \in \Lambda(q)$ if and only if $q(\theta) \in [0, \bar{q}]$, for all θ.

Private goods: Private goods have the same consumption sets as excludable public goods but have a different cost structure because of rivalry. The cost is now increasing in aggregate consumption $E[q(\theta)]$ and we therefore take $K(q)$ to be equal to $k(E[q(\theta)])$.

We impose the assumption that the provision rule q is continuously differentiable. This is a technical assumption that facilitates the characterization of admissible policies.[13]

Surplus-maximization. We say that an admissible policy is surplus-maximizing or first-best if the budget constraint in (1) holds as an equality and the provision-rule q is chosen so as to maximize

$$S(q) := E[v(\theta, q(\theta))] - K(q) \; .$$

We denote the surplus-maximizing provision rule by q^*. We say that a policy $p = (q, c, G)$ is surplus-maximizing if (1) holds as an equality and $q = q^*$.

[12] This assumption simplifies our proof. Without the assumption there would be an extra step. We would then have to show that unbounded provision levels are incompatible with the economy's resource constraint.

[13] As shown in the Online-Appendix, with a discrete set of types, there is no need of such an assumption.

Welfare. An admissible policy $p = (q, c, G)$ confronts individuals with a randomized mechanism. We denote by

$$U(\theta \mid p) := \int_{\mathbb{R}_+} \Phi\left(x + c(\theta) + v(\theta, q(\theta))\right) dG(x) \ ,$$

the expected utility that an individual with type θ realizes under such a policy. To compute the welfare induced by a policy p, we use a welfare function

$$W(p) = E[\gamma(\theta) U(\theta \mid p)] \ ,$$

where $\gamma : \Theta \to \mathbb{R}_+$ is a function that specifies the welfare weights for different types of individuals. The function Φ is assumed to be concave and increasing. It admits two different interpretations. First, in a model with randomized outcomes, Φ may capture the risk attitude of individuals. If the function Φ is strictly concave, then individuals are risk-averse. If Φ is linear, then individuals are risk-neutral. Second, and independently of whether randomized outcomes are involved, the concavity of Φ can be viewed as a measure of a welfare-maximizer's inequality aversion. We refer to the special case in which Φ is linear and $\gamma(\theta) = 1$, for all θ as unweighted utilitarian welfare. In this case, $W(p)$ equals $\int_{\mathbb{R}_+} x \ dG(x) + E[c(\theta) + v(\theta, q(\theta))]$. We denote the set of welfare-maximizing policies by P^W, i.e. $p \in P^W$ if there exist functions γ and Φ so that p maximizes $E[\gamma(\theta) U(\theta \mid p)]$ over the set of admissible policies.

3.2 Political competition

There are two politicians. A policy for politician $j \in \{1, 2\}$, is an admissible triple $p^j = (q^j, c^j, G^j)$. We assume that the two politicians choose policies simultaneously and independently. In particular, the favors offered to different voters are also drawn simultaneously and independently. This implies that politician 1 cannot see the favors offered by politician 2 and then generate a majority by offering less too a tiny group of voters and offering more to anybody else.

Voters observe the favors offered to them by the two politicians and then caste their vote, i.e. voter i observes, for each politician j, the provision rule q^j, the consumption rule c^j and his drawing from the distribution G^j, henceforth denoted by x_i^j.[14] Voter i votes for politician 1 if $u(\theta_i \mid q^1, c^1) + x_i^1 > u(\theta_i \mid q^2, c^2) + x_i^2$, tosses a coin if these expressions are equal, and votes for politician 2 otherwise. If the distributions G^1 and G^2 are atomless, the probability that any one voter i votes for politician 1 is given by

$$\Pi^1(p^1, p^2) := E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 + u(\theta_i \mid q^1, c^1) - u(\theta_i \mid q^2, c^2)\right) dG^1\left(x_i^1\right)\right]. \tag{2}$$

By the law of large numbers, we can also interpret $\Pi^1(p^1, p^2)$ as politician 1's vote share and $\Pi^2(p^1, p^2) = 1 - \Pi^1(p^1, p^2)$ as politician 2's vote share.

[14] Note that risk attitudes play no role for the characterization of voting behavior because individuals caste their votes after having seen their outcomes from the politicians' lotteries.

Definition 1 *Two policies* p^1_{eq} *and* p^2_{eq} *are a Nash equilibrium if* $\Pi^1(p^1_{eq}, p^2_{eq}) \geq \Pi^1(p^1, p^2_{eq})$, *for every admissible* p^1, *and* $\Pi^2(p^1_{eq}, p^2_{eq}) \geq \Pi^2(p^1_{eq}, p^2)$, *for every admissible* p^2.

3.3 The main results

The following two theorems state our main results. We provide a complete equilibrium characterization in Corollary 1 below. The proofs of the theorems are in the Appendix. Most of our analysis focuses on equilibria in pure strategies, i.e. we do not consider the possibility that politicians randomize over various admissible policies. Equilibrium existence can therefore not be guaranteed with an appeal to standard results. We provide conditions for the existence of pure strategy equilibria in Section 6 below.

Theorem 1 *If the set of Nash equilibria is non-empty, then there is one and only one symmetric Nash equilibrium. In this equilibrium, policies are surplus-maximizing.*

Theorem 2 *Suppose that individuals are risk averse. There is a surplus-maximizing policy* p^* *so that* $\Pi^1(p^*, p^W) \geq \frac{1}{2}$, *for all* $p^W \in P^W$. *Moreover, the inequality is strict whenever* p^W *is not surplus-maximizing.*

Theorem 1 can be viewed as a first welfare theorem for the given model of political competition. Provided that there is an equilibrium at all, the unique symmetric equilibrium is surplus-maximizing and therefore ex-post efficient. Due to the assumption that preferences are quasi-linear, the setup is more restrictive than the one in which the first welfare theorem for competitive equilibrium allocations holds. Still, it is intriguing to note that the first welfare theorem for competitive equilibrium allocations no longer applies if there are public goods or externalities. Theorem 1, by contrast, applies irrespectively of whether the publicly provided goods are private or not. Hence, in the given model, political competition generates Pareto-efficient outcomes in all circumstances, whereas competitive markets generate Pareto-efficient outcomes only if the goods involved are private. This positive assessment, however, depends on the assumption that the only policy goal is to get an ex-post efficient outcome. If the policy goal is to maximize welfare, then, Theorem 2 tells us that this is incompatible with a political equilibrium.

Theorem 2 shows that there is a surplus-maximizing policy that defeats *any* welfare-maximizing one, i.e. a politician who knows that his opponent runs on some welfare-maximizing platform, can make sure that he will not be defeated. Thus, the only Pareto-efficient outcome that is compatible with a political equilibrium is the surplus-maximizing one.

Theorem 2 comes as a surprise if one starts out with the basic intuition that the vote share that is generated by a welfare-maximizing policy should depend on the distribution of types. For a pure public-goods application, suppose that the distribution F is such that more than half of the electorate has type $\underline{\theta}$. Hence, there is a majority of voters who do not value the public good at all. Now suppose a politician proposes a policy that maximizes a

welfare-function that assigns a positive weight only to this group of individuals. As follows from Theorem 2, a politician who targets this big group of voters with a low public-goods preference will lose against a politician who proposes the surplus-maximizing quantity in combination with pork-barrel spending. The latter will win a certain fraction of the voters in the majority group because of the favors he offers to them, and, in addition, he will get the votes of the minority group. The overall effect is that he will win more than fifty percent of the votes. This policy is successful only because voters have private information on their preferences. This forces a politician who targets the big group to respect incentive compatibility constraints which makes it difficult to channel resources to the big group only. If the deal for the big group became too good, incentive compatibility would fail because individuals from the small group would declare that they also belong to the big group.

In Section 4 below, we provide a more detailed explanation of the logic that underlies Theorems 1 and 2. Our discussion will also clarify how a complete equilibrium characterization is obtained, and, in particular, what pork-barrel spending looks like in equilibrium. The results from that equilibrium characterization are summarized in following Corollary.

Corollary 1 *If the set of equilibria is non-empty, then the unique symmetric equilibrium* $p_{eq} = (q_{eq}, c_{eq}, G_{eq})$ *is such that:*

(a) The provision rule is surplus-maximizing $q_{eq} = q^*$.

(b) Private goods consumption is such that for any $\theta \in \Theta$,

$$c_{eq}(\theta) = v(\bar{\theta}, q^*(\bar{\theta})) - v(\theta, q^*(\theta)) - \int_{\theta}^{\bar{\theta}} v_1(s, q^*(s))ds \ , \tag{3}$$

(c) G_{eq} *is uniform on* $[0, 2(e + S_v(q^*))]$, *where*

$$S_v(q^*) := S(q^*) - \left(v(\bar{\theta}, q^*(\bar{\theta})) - E\left[\frac{F(\theta)}{f(\theta)} v_1(\theta, q^*(\theta)) \right] \right) \ . \tag{4}$$

is an expression that is also known as the virtual surplus from public-goods provision.

Our analysis focusses on symmetric equilibria. By Theorem 1 if there is a pure strategy equilibrium at all, then there is also a symmetric one. A priori, however, we cannot rule out the existence of asymmetric equilibria. Such equilibria exist if and only if there is a deviation from the equilibrium policy in which both G is different from G_{eq} and q is different from is q^*, and which yields exactly a vote share of one half. We discuss such double deviations in more detail in Section 6 on the existence of equilibria. There we show that pure strategy equilibria exist if and only if such double deviations cannot generate more than fifty percent of the votes. Thus, the existence of an asymmetric equilibrium is a knife-edge case for the existence of an equilibrium in pure strategies.

4 The underlying mechanism

To obtain an intuitive understanding of our main results we will in the following provide heuristic answers to the following questions: Why do vote-share-maximizers behave as surplus-maximizers? Why do welfare-maximizers lose elections? Why do politicians who maximize pork-barrel spending lose elections? Why doesn't it payoff to propose the median voter's preferred policy? This discussion complements the proof in the Appendix which takes care of all formal details.

4.1 How vote-share maximizers become surplus-maximizers

In our model, individuals differ in their types, i.e. they have different views on the desirability of public-goods provision. Some are willing to give up a lot of private goods consumption in exchange for increased public goods provision, others are not willing to give up anything. The cross-section distribution of these preferences is known to everybody, while the preferences of a particular individual are privately observed only.

Now, consider a race between two politicians who seek to get the support of a majority of individuals. At first glance, one might think that they will propose policies that are attractive to a majority of "types". For instance, in a model with only two types one might expect a political equilibrium in which both politicians propose the policy that maximizes the utility of the bigger group, with no concern for the utility of the smaller group. This would indeed be true if there was no possibility of pork-barrel spending.[15]

So, to get an intuitive understanding of Theorems 1 and 2, one has to clarify how pork-barrel spending turns vote-share-maximizing politicians into surplus-maximizing ones. Our explanation is based on three observations.

Observation 1: Politician 1 chooses welfare weights for politician 2, and vice versa. We argue that politicians can be thought of as maximizing a welfare function with particular weights that are determined in the political process. We first clarify the notion of a welfare weight that is of frequent use in normative public economics.[16] Consider a welfare function

$$W(p) = E\left[\int_{\mathbb{R}_+} \gamma(\theta)\Phi(x + u(\theta \mid q, c))\, dG(x)\right] ,$$

and denote by $p^W = (q^W, c^W, G^W)$ the corresponding welfare-maximizing policy. Welfare-weights provide us with a characterization of p^W that highlights how a maximizer of W trades off the well-being of different types of individuals. The welfare weight of individuals

[15] See Bierbrauer and Boyer (2013) for a proof of this assertion in the context of a two-type-model of non-linear income taxation.

[16] For a more general discussion, see Saez and Stantcheva (2013).

with type θ is the marginal welfare gain that is realized if the utility $u(\theta \mid q, c)$ is slightly increased. It is given by

$$w^*(\theta) := \int_{\mathbb{R}_+} \gamma(\theta)\Phi'(x + u(\theta \mid q^W, c^W))\, dG^W(x)\ .$$

Now, the welfare-maximizing public goods mechanism (q^W, c^W) has the following property: It maximizes an additive welfare objective

$$E[w^*(\theta)\ u(\theta \mid q, c)]$$

over the set of admissible allocations, with G fixed at G^W.[17] That is, a maximizer of W behaves as if pork-parrel spending was exogenously given at G^W, and the remaining problem was to find a welfare-maximizing mechanism for public goods provision, based on an additive welfare function in which the weights are given by the function $w^* : \Theta \to \mathbb{R}_+$.

We now use this approach to determine the welfare weights that are consistent with the behavior of politician 1. We first ask how her vote-share increases as the utility of individuals with type θ is slightly increased. This gives rise to a marginal gain of votes that equals

$$\pi^1(\theta \mid p^1, p^2) := \int_{\mathbb{R}_+} g^2\left(x_i^1 + u(\theta \mid q^1, c^1) - u(\theta \mid q^2, c^2)\right)\ dG^1\left(x_i^1\right)\ , \tag{5}$$

where g^2 is the derivative of G^2. The marginal vote-shares that arise in a symmetric political equilibrium are denoted by $\pi^1_{eq}(\theta) := \pi^1(\theta \mid p_{eq}, p_{eq})$, where $p_{eq} = (q_{eq}, c_{eq}, G_{eq})$ is the equilibrium policy. In analogy to our reasoning on the role of welfare weights in normative public economics, we can think of politician 1 as choosing (q^1, c^1) to maximize

$$E[\pi^1_{eq}(\theta)\ u(\theta \mid q, c)]$$

over the set of admissible allocations, with G^1 fixed at G_{eq}. Thus, in choosing a public goods mechanism, politician 1 behaves as if she was maximizing an additive welfare function in which the weight of a type θ-voter is given by $\pi^1_{eq}(\theta)$.

We will now argue that G_{eq} is a uniform distribution. As is apparent from (5), the consequence is that $\pi^1_{eq}(\theta)$ is same for all types θ so that politician 1 simply maximizes an unweighted sum of utilities, $E[u(\theta \mid q, c)]$.

Observation 2: G_{eq} is uniform. In a symmetric equilibrium politicians 1 and 2 choose the same mechanism for public goods provision, which implies that $u^1(\theta \mid q^1, c^1) = u^2(\theta \mid q^2, c^2)$, for all θ. Politician 1's vote share then equals

$$\Pi^1(p^1, p_{eq}) \quad = \quad \int_{\mathbb{R}_+} G^2\left(x_i^1\right) dG^1\left(x_i^1\right). \tag{6}$$

[17] Mathematically, maximizing a concave welfare function is equivalent to maximizing an additive welfare function that has the same slope at the welfare-maximizing policy p^W.

With the public goods mechanisms held fix, she chooses G^1 so as to maximize this expression subject to $\int_0^\infty x_i^1 \; dG^1(x_i^1) \leq e - E[c^1(\theta)] - K(q^1)$. Politician 2 solves the same problem.

This is very similar to the divide-the-dollar-game in Myerson (1993), even though Myerson's setup is different at first glance: There is no public-goods provision and there is no privacy of information. All voters are endowed with e dollars, and a policy is simply a way of reshuffling these dollars in the electorate. Formally, a policy is characterized by a distribution function G over the positive reals which has to satisfy the resource constraint $\int_{\mathbb{R}_+} x_i \; dG(x_i) \leq e$. Vote shares are, again, given by (6). Myerson (1993) shows that there is one and only one symmetric equilibrium, a uniform distribution on $[0, 2e]$. It is easy to verify that this is an equilibrium:[18] If politician 2 follows the equilibrium strategy, then politician 1's vote share when choosing some arbitrary strategy G^1 equals

$$\int_{\mathbb{R}_+} G^2\left(x_i^1\right) dG^1\left(x_i^1\right) \;=\; \int_{\mathbb{R}_+} \min\left\{\tfrac{x_i^1}{2\,e}, 1\right\} dG^1\left(x_i^1\right) \;\leq\; \int_{\mathbb{R}_+} \tfrac{x_i^1}{2\,e} dG^1\left(x_i^1\right) \;=\; \tfrac{1}{2}\,,$$

where the inequality is strict if and only if G^1 assigns positive mass to $(2e, +\infty)$, and the last equality uses the budget constraint, $\int_0^\infty x_i^1 \; dG^1(x_i^1) = e$. Since $\frac{1}{2}$ is the vote share obtained by distributing pork according to a uniform distribution on $[0, 2e]$, there is no deviation that increases politician 1's vote-share.

The divide-the-dollar-game in Myerson (1993) and the problem to distribute pork with the mechanisms for public goods provision held fix are equivalent except that pork-barrel spending in Myerson is bounded by the fixed dollar endowment e, and in our analysis it is bounded by that part of the endowment that is left over once the resource requirement of the public goods mechanism has been taken into account, i.e. by $e - E[c^1(\theta)] - K(q^1)$. Thus, the equilibrium distribution of pork is uniform on $[0, 2e]$ in Myerson (1993), and uniform on $[0, 2\,(e - E[c^1(\theta)] - K(q^1))]$ in our analysis.[19]

Observation 3: G_{eq} uniform turns a vote-share maximizer into a surplus-maximizer. Taken together Observations 1 and 2 imply that the mechanism for public goods provision that arises in a political equilibrium maximizes an unweighted sum of utilities $E[u(\theta \mid q, c)]$. It remains to be shown that this leads to surplus-maximizing outcomes. This would be obvious if there was no requirement of incentive compatibility.[20]

[18] Myerson (1993) also provides a very elegant proof that this is the only symmetric equilibrium. It uses variational arguments to show that under any other allocation of pork-barrel spending one politician would have an incentive to deviate, thereby winning a majority of votes.

[19] The requirement of incentive compatibility can be shown to imply that $S_v(q^1) = -(E[c^1(\theta)] + K(q^1))$. Hence, pork-barrel spending is uniform on $[0, 2(e + S_v(q^1))]$ as stated in Corollary 1. This step is more technical, albeit involving standard arguments, and therefore left to the Appendix.

[20] With G fixed at G_{eq}, the budget constraint implies $E[c^1(\theta)] = e - K(q^1) - \int_{\mathbb{R}_+} x \; dG_{eq}(x)$. Substituting this into the objective yields

$$E[u(\theta \mid q, c)] = E[c(\theta)] + E[v(\theta, q(\theta))] = E[v(\theta, q(\theta))] - K(q) + e - \int_{\mathbb{R}_+} x \; dG_{eq}(x)\,,$$

Here, however, we are in a second-best environment as policies have to be resource-feasible and incentive-compatible. Thus, we need to show that a maximizer of $E[u(\theta \mid q, c)]$ chooses a second-best policy that is also a first-best policy, i.e. a policy that cannot be Pareto-improved upon in the set of policies that only have to satisfy the economy's resource constraint.

We begin with a clarification of how the requirement of incentive compatibility restricts the set of admissible policies.[21] First, the requirement of incentive compatibility eliminates any degrees of freedom in the choice of the private goods consumption schedule c. It implies that

$$c(\theta) = v(\bar{\theta}, q(\bar{\theta})) - v(\theta, q(\theta)) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds \ , \tag{7}$$

so that the function c can be derived from (7) if q is given. In the following, we will therefore represent an admissible policy as a pair $p = (q, G)$ with the understanding that the corresponding c-function then follows from (7). Second, the requirement of incentive compatibility also implies that individuals with higher types realize an information rent given by $v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds$.[22] For later reference, note that the expected value of the information rent is given by

$$E\left[v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds\right] = v(\bar{\theta}, q(\bar{\theta})) - E\left[\frac{F(\theta)}{f(\theta)} v_1(\theta, q(\theta))\right] \ . \tag{8}$$

Third, the resource constraint becomes

$$\int_0^{\infty} x \ dG(x) \leq e + S_v(q) \ , \tag{9}$$

where

$$S_v(q) := E[v(\theta, q(\theta))] - K(q) - \left(v(\bar{\theta}, q(\bar{\theta})) - E\left[\frac{F(\theta)}{f(\theta)} v_1(\theta, q(\theta))\right]\right) \tag{10}$$

is often referred to as the *virtual* surplus. An inspection of equations (9), (10) and (8) reveals that information rents reduce the resources that are available for pork-barrel spending: $\int_{\mathbb{R}_+} x \ dG(x)$ is bounded by $e + S_v(q)$. If there were no information rents to be paid it would be bounded by the (non-virtual) surplus $S(q)$.

To understand the choice of a politician who maximizes an unweighted utilitarian welfare function, we first consider the maximization of a more general welfare objective

$$W(p) = E\left[\gamma(\theta) \int_0^{\infty} \Phi\left(x + v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds\right) dG(x) \ ,\right]$$

an expression that is, up to an additive constant, equal to the surplus from public-goods provision, $S(q) = E[v(\theta, q(\theta))] - K(q)$.

[21] Lemma A.1 in the Appendix provides a characterization.

[22] For a given amount of pork x, this expression is equal to the difference between the lowest type's utility level and the utility of type θ.

subject to the resource constraint in (9). Since Φ is a non-convex function, Jensen's inequality implies that there is a solution to this problem so that G is a degenerate distribution with unit mass at $e + S_v(q)$. Hence, the remaining problem is to choose q so as to maximize

$$E\left[\gamma(\theta)\ \Phi\left(e + S_v(q) + v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds\right)\right] \tag{11}$$

subject to (9). The expression in (11) can be interpreted as follows: There is a base level of utility, $e+S_v(q)$, that every voter gets and an information rent $v(\bar{\theta}, q(\bar{\theta}))-\int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds$ that is realized only by voters with type $\theta > \underline{\theta}$. We now argue that the difference between a weighted and an unweighted welfare function can be attributed to differences in the weighting of the common base utility relative to the information rents. For simplicity, we assume in the following that Φ is linear and that the average welfare weight equals 1, $E[\gamma(\theta)] = 1$.[23] Using the definition of the virtual surplus in (10), we can then write welfare as

$$e + S(q) + E\left[\frac{F(\theta)}{f(\theta)} v_1(\theta, q(\theta))\right] - E\left[\gamma(\theta) \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds\right] ,$$

or, after an integration by parts, as

$$e + S(q) - E\left[(\Gamma(\theta) - 1)\ \frac{F(\theta)}{f(\theta)}\ v_1(\theta, q(\theta))\right] ,$$

where $\Gamma(\theta) =: \int_{\underline{\theta}}^{\theta} \gamma(s)\ \frac{f(s)}{F(\theta)}\ ds = E[\gamma(s) \mid s \leq \theta]$ is the average weight among individuals with a type below θ. If γ is a strictly decreasing function then $\Gamma(\theta) \geq 1$, for all θ, with a strict inequality whenever $\theta < \overline{\theta}$. By contrast, if all weights are equal so that $\gamma(\theta) = \Gamma(\theta) = 1$, for all θ, the objective boils down to surplus-maximization, $e + S(q)$.

This shows that the difference between weighted and unweighted welfare is a difference in the evaluation of information rents. For an unweighted welfare function they have a benefit – more utility for higher types – and a cost – less base utility – which cancel each other exactly. An inequality-averse welfare-maximizer, by contrast, cares about the distribution of the benefits from publicly provided goods. He is therefore putting less weight to the benefits of information rents as those are mostly realized by the individuals with high types. Maximizing an inequality-averse welfare function will therefore give rise to downward distortions of public-goods supply relative to the surplus-maximizing level. These downward distortions make it possible to increase the lowest type's utility at the expense of information rents that are realized by individuals with higher types.[24] The maximizer of unweighted welfare function, by contrast, has no desire to limit information rents and does not deviate from surplus-maximizing public goods provision.

[23] With degenerate pork-barrel spending, assuming that Φ is concave has similar implications as assuming that Φ is linear and that γ is decreasing. The literature refers to "Φ concave" as a specification with endogenous welfare weights and to "Φ linear and γ decreasing" as a specification with exogenous welfare weights. Both imply a desire to redistribute from high to low types.

[24] See, for instance, Ledyard and Palfrey (1999) and Hellwig (2005).

4.2 Alternative objectives

We have shown that with uniform pork-barrel spending, vote-share maximizers are turned into surplus-maximizers. Consequently, a politician who departs from surplus-maximizing public goods provision does not reach a maximal vote-share and will lose the election if the opponent behaves according to the surplus-maximizing equilibrium policy p_{eq}. Still, it is interesting to illustrate what an alternative objective would imply for the distribution of votes among the two politicians. In the following, we discuss some specific alternatives: Which voters would support the policy offered by an inequality-averse welfare-maximizer? Who would vote for a politician who maximizes the size of the pork-barrel? Who would support a politician who proposes the median voter's preferred policy?

Inequality-aversion. In comparison to a surplus-maximizer, inequality-averse welfare-maximizers generate more base utility and lower information rents. Thus, when competing against a surplus-maximizer in an election, a welfare-maximizer will realize a higher vote share among individuals with low types who hardly benefit from information rents and a lower vote share among individuals with high types who realize significant information rents. The increased vote-share among low type individuals, however, does not compensate for the loss of votes among high type individuals. By Theorem 2, the net effect is a loss of votes.

The median voter's preferred policy. We now consider a welfare function with $\gamma(\theta) = 1$ if $\theta = \theta^m$ where $F(\theta^m) = \frac{1}{2}$ and $\gamma(\theta) = 0$ if $\theta \neq \theta^m$. Again, we assume for simplicity that Φ is linear. Consequently, welfare can be written as

$$e + S_v(q) + v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta^m}^{\bar{\theta}} v_1(s, q(s))ds \ .$$

This welfare function gives rise to an equal weighting of the common base utility, $e+S_v(q)$, and the information rent realized by one particular type of voter, namely the median type, $v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta^m}^{\bar{\theta}} v_1(s, q(s))ds$. The information rents of all other types of voters receive no weight in this objective function. The mechanism which maximizes this objective function has been characterized by Röell (2012) and Brett and Weymark (2014).[25] Relative to the surplus-maximizing outcome, there are downward distortions of public goods supply for types above the median. These distortions stem from the desire to redistribute from high types to the median type. Analogously, there are upward distortion of public goods supply for types below the median because of the desire to redistribute from low types

[25] Röell (2012) and Brett and Weymark (2014) study an alternative model of political competition, the citizen-candidate model due to Osborne and Slivinski (1996) and Besley and Coate (1997). This reduces the policy domain to welfare-maximizing policies which assign weight to only one particular voter type. Röell (2012) and Brett and Weymark (2014) show that the median voter's preferred policy is a Condorcet winner in this policy domain. They develop their argument in the context of a model of income taxation, but the analysis extends to the given setup with publicly provided goods.

to the median type. By our Theorem 2, the median voter's preferred policy is defeated by the equilibrium policy p_{eq}. Proposing the median voter's preferred policy generates more support by types close the median, at the cost of reduced support by high and low types. By Theorem 2, the net effect is that a politician who maximizes the median voter's welfare will be defeated.

Maximal pork-barrel spending. Politician 1's budget constraint is given by

$$\int_0^\infty x_i^1 \, dG(x_i^1) \leq e + S_v(q^1).$$

Maximizing the size of the pork-barrel therefore requires to maximize $e + S_v(q^1)$. Our previous discussion has shown that this is equivalent to maximizing a particular welfare function that weights only the base utility that any one voter gets and which assigns no weight to the information rents that are realized by voters with high types. Maximizing the size of the pork-barrel is hence equivalent to maximizing a Rawlsian welfare function that assigns weight only to the worst-off type. We have previously argued that all welfare-maximizers lose elections. Since a politician who maximizes the size of the pork-barrel is a welfare-maximizer, this type of politician will lose as well.

5 Income taxation

In this section we extend the analysis to a Mirrleesian model of income taxation, under the maintained assumption that preferences are quasi-linear in private goods consumption.

5.1 The economic environment

Preferences. Voter i's utility function is given by $u_i = c_i - \tilde{v}(\omega_i, y_i)$. As before, c_i denotes the voter's consumption of private goods. The voter's contribution to the economy's output is denoted by y_i. The literature on income taxation refers to c_i also as i's *after-tax-income* and to y_i as i's *pre-tax-income*. The function $\tilde{v}$ captures the utility cost of productive effort. This cost depends on how much output is generated and on the individual's type ω_i, which belongs to a compact set of types $\Omega = [\underline{\omega}, \overline{\omega}]$. We assume that the cost function is increasing and convex in the level of output so that $\tilde{v}_2(\omega_i, y_i) > 0$ and $\tilde{v}_{22}(\omega_i, y_i) > 0$, for all $\omega_i \in \Omega$ and all $y_i > 0$. Costs are zero if no output is generated, so that $\tilde{v}(\omega_i, 0) = 0$, for all ω_i. Higher types have lower absolute and marginal effort costs, i.e. $\tilde{v}_1(\omega_i, y_i) < 0$, and $\tilde{v}_{12}(\omega_i, y_i) < 0$, for all $\omega_i \in (\underline{\omega}, \overline{\omega})$ and all $y_i > 0$.[26] Finally, the

[26] In the literature, one often finds additional assumptions which are not needed for our purpose. Specifically, the function $\tilde{v}$ is often derived as follows: It is assumed that there is a disutility $\tilde{v}(h_i)$ of having to work h_i hours. This disutility is assumed to be the same for all individuals. By contrast, individuals differ in their hourly wages. Hence, if an individual with wage ω_i wants to achieve an income of y_i she has to work for $\frac{y_i}{\omega_i}$ hours and this comes with a utility cost equal to $\tilde{v}\left(\frac{y_i}{\omega_i}\right)$.

Inada conditions hold: For all $\omega_i \in \Omega$, $\lim_{y_i \to 0} v_2(\omega_i, y_i) = 0$, and $\lim_{y_i \to \infty} v_2(\omega_i, y_i) = \infty$. Any one individual's skill type ω_i is taken to be a random variable with a cumulative distribution function F and a continuous density f.

Policies. A policy $p = (y, c, G)$ consists of (i) a function $y : \Omega \to \mathbb{R}_+$ which determines the individual's contribution to the economy's output, (ii) a function $c : \Omega \to \mathbb{R}_+$ which determines any one individual's private goods consumption as a function of the individual's type and (iii) a cross-section distribution of lump-sum-transfers G. If individual i receives a draw x from the distribution G, then the individual's private goods consumption equals x in the event that $\omega_i = \underline{\omega}$ and equals $x + c(\omega)$ if $\omega_i = w$. In the income tax model, incentive compatibility will imply that individuals with higher types provide (weakly) more output and therefore have to be compensated by (weakly) higher consumption. Consequently, c will be a non-decreasing function. We therefore impose the normalization that $c(\underline{\omega}) = 0$. As the support of G is bounded from below by 0 this ensures that non-negativity constraints on consumption levels will not be violated.

An admissible policy has to satisfy the following incentive compatibility constraints: For all ω and ω',

$$c(\omega) - \tilde{v}(\omega, y(\omega)) \geq c(\omega') - \tilde{v}(\omega, y(\omega')) \ .$$

In addition, policies have to be feasible which requires that

$$\int_0^\infty x \ dG(x) + E[c(\omega)] \leq e + E[y(\omega)] \ . \tag{12}$$

Finally, we assume that the time that individuals can devote to the generation of income is bounded. For every type ω, there is an upper bound $\bar{y}(\omega)$. Hence $y : \Omega \to \mathbb{R}_+$ can be part of an admissible policy only if, for all ω, $y(\omega) \leq \bar{y}(\omega)$.

Taxation Principle and marginal income tax rates. According to the *Taxation Principle*, see Hammond (1979) and Guesnerie (1995), there is an equivalence between allocations that are admissible and allocations that are decentralizable via an income tax system. For the given setting, this implies that, to any admissible allocation, there exists an income tax function $T : y \to T(y)$ so that

$$y(\omega) \quad \in \quad \text{argmax}_{y' \in \mathbb{R}_+} \quad y' - T(y') - \tilde{v}(\omega, y') \ .$$

Consequently, to implement an admissible policy, a policy-maker can specify an income tax function, and then let individuals choose a utility-maximizing level of income. If the latter approach is taken, we can infer the marginal tax rates that are associated with any income level that lies in the image of the income schedule $y : \Omega \to \mathbb{R}_+$. Assuming that the solution of the household problem above can be characterized by a first-order condition, we obtain

$$T'(y(\omega)) = 1 - \tilde{v}_2(\omega, y(\omega)) \ . \tag{13}$$

Hence, given an income schedule $y : \Omega \to \mathbb{R}_+$ that is part of an admissible policy, we can use equation (13) to trace out the corresponding marginal income tax rates.

Surplus-maximization. We say that a policy is surplus-maximizing or first-best if the budget constraint in (12) holds as an equality and the provision-rule y is chosen so as to maximize

$$S(y) := E[y(\omega)] - E[\tilde{v}(\omega, y(\omega))]$$

subject to the constraint that $y(\omega) \leq y(\bar{\omega})$, for all ω. We denote the surplus-maximizing provision rule by y^*. We say that a policy $p = (y, c, G)$ is surplus-maximizing if (12) holds as an equality and $y = y^*$.

Welfare. We denote by

$$U(\omega \mid p) := \int_0^\infty \Phi\left(x + c(\omega) - \tilde{v}(\omega, y(\omega))\right) dG(x) \ ,$$

the expected utility that an individual with type ω realizes under a policy $p = (y, c, G)$. Again, the function Φ is assumed to be concave and increasing. To compute the welfare induced by a policy p, we use a welfare function $W(p) = E[\gamma(\omega)U(\omega \mid p)]$, where $\gamma : \Omega \to \mathbb{R}_+$ specifies the welfare weights for different types of individuals.

5.2 The main results reconsidered

Our main results in Theorems 1 and 2 extend to the given setting. By Theorem 1, equilibrium policies are surplus-maximizing. Here, this means that, for almost all $\omega \in \Omega$, $1 = \tilde{v}_2(\omega, y^*(\omega))$. Hence, in a political equilibrium, there is no use of distortionary taxation, so that almost every voter faces a marginal tax rate of 0. By the equilibrium characterization in Corollary 2 below, this does not mean that there is no redistribution. Politicians engage in redistribution. However, they do so by means of pork-barrel spending, and not by making use of type-dependent income taxation.

By Theorem 2, there is a surplus-maximizing policy p^* that defeats any welfare-maximizing policy. As we elaborate in more detail below, the literature on optimal income taxation has focussed on welfare-maximization as the policy-objective. By Theorem 2, no such policy can emerge in a political equilibrium.

We do not provide separate proofs for the model of income taxation. Such proofs would, with some changes in details, reiterate the arguments developed in the model of publicly provided goods. We do however provide an equilibrium characterization. Accordingly, equilibrium policies involve first-best output provision according to y^* and transfers which are drawn from a uniform distribution on $[0, 2(e + S_v(y^*))]$, where $S_v(y^*)$ is the *virtual surplus* associated with output provision rule y^*.

Corollary 2 *Suppose that $\Omega = [\underline{\omega}, \overline{\omega}]$. If the set of equilibria is non-empty, then the unique symmetric equilibrium $p_{eq} = (y_{eq}, c_{eq}, G_{eq})$ is such that:*

(a) Before-tax-incomes are surplus-maximizing $y_{eq} = y^*$.

(b) Private goods consumption is such that for any $\omega \in \Omega$,

$$c_{eq}(\omega) = \tilde{v}(\omega, y^*(\omega)) - \tilde{v}(\underline{\omega}, y^*(\underline{\omega})) - \int_{\underline{\omega}}^{\omega} \tilde{v}_1(s, y^*(s))ds \ , \tag{14}$$

(c) G_{eq} is uniform on $[0, 2(e + S_v(y^*))]$, where

$$S_v(y^*) := S(y^*) + \tilde{v}(\underline{\omega}, y^*(\underline{\omega})) + E\left[\frac{1 - F(\omega)}{f(\omega)} \tilde{v}_1(\omega, y^*(\omega))\right] \ . \tag{15}$$

5.3 Political failures

The literature on optimal income taxation considers the problem of choosing a tax policy $p = (y, c, G)$ so as to maximize a welfare function $W(p)$. Typically it is assumed that Φ is strictly concave so that individuals are risk averse. Then, under a welfare-maximizing policy the lottery G is degenerate and assigns unit mass to $e+S_v(y)$. Thus, we can identify pork-barrel spending according to G_{eq} as a first political failure. The remaining question then is how the income schedule that is chosen in a political equilibrium relates to the welfare-maximizing one. Welfare-maximization requires to choose y so as to maximize

$$E\left[\gamma(\omega)\Phi\left(e + S_v(y) - \tilde{v}(\underline{\omega}, y(\underline{\omega})) - \int_{\underline{\omega}}^{\omega} \tilde{v}_1(s, y(s))ds\right)\right]$$

subject to the constraint that y is non-decreasing. The standard way of solving this problem is to start with a relaxed problem in which the monotonicity constraint is not taken into account, and then to check under which conditions the solution to the relaxed problem is monotonic. The relaxed problem yields the following first order conditions, which characterize the welfare maximizing income schedule y^W: for any type ω, $y^W(\omega)$ solves

$$T'(y(\omega)) := 1 - v_2(\omega, y(\omega)) = -\frac{1 - F(\omega)}{f(\omega)} \left(1 - \Gamma(\omega)\right) \tilde{v}_{12}(\omega, y(\omega)) \ , \tag{16}$$

where the expression

$$\Gamma(\omega) := \frac{E[\gamma(s)\Phi'(\cdot) \mid s \geq \omega]}{E[\gamma(s)\Phi'(\cdot) \mid s \geq \underline{\omega}]} \ ,$$

gives the average welfare weight of individuals with type ω and higher relative to the average individual. A standard result in the literature on optimal income taxation (see, e.g. Hellwig, 2007) is that optimal marginal tax rates are strictly positive, except possibly at the top and the bottom of the type distribution. In the given framework, one can use equation (16) to verify these results. Since $\tilde{v}_{12}(\omega, y) < 0$, for all ω and y, and, under

any admissible policy, $\Gamma(\omega) < 1$, for all $\omega \geq \underline{\omega}$, it follows that $T'(y(\omega)) > 0$, for all $\omega \in (\underline{\omega}, \overline{\omega})$.[27] If we relate this observation to Theorem 1, we identify another political failure: In the political equilibrium marginal tax rates are equal to 0 throughout, whereas under a welfare-maximizing policy they should be positive, except at the top and the bottom. An inequality-averse welfare-maximizing policy uses distortionary taxation so as to redistribute between highly productive and less productive individuals. In a political equilibrium, this type of redistribution does not take place.

To sum up, from a welfare-perspective, political equilibria give rise to undesirable redistribution via pork-barrel spending, whereas the scope for desirable redistribution via type-dependent income taxation remains unused.

6 Remarks on equilibrium existence

Our model of political competition has a multi-dimensional policy domain. As is well known, for such games, the existence of a pure strategy equilibrium – here, an equilibrium in which each politician chooses one admissible policy with probability 1 – cannot be taken for granted. In the following, we first state a necessary and sufficient condition for the existence of a pure strategy equilibrium. We will then show how to apply this condition in the context of our model. Second, if a pure strategy equilibrium does not exist, this raises the question whether mixed strategy equilibria admit a meaningful interpretation. We will comment on the existence and the interpretation of mixed strategy equilibria in our setting and discuss their welfare implications. Specifically, we will argue that mixed strategy equilibria give rise to outcomes that are not Pareto-efficient and can hence be viewed as political failures.

6.1 Pure strategy equilibrium

The following corollary states a necessary and sufficient condition for the existence of a Nash equilibrium in pure strategies.

Corollary 3 *An equilibrium exists if and only if there is no admissible policy p with $\Pi^1(p, p_{eq}) > \frac{1}{2}$, where p_{eq} is the policy which has been characterized in Corollaries 1 and 2.*

To see that Corollary 3 is true, note first that, by Theorem 1, if the set of pure-strategy equilibria is non-empty, then (p_{eq}, p_{eq}) is a Nash equilibrium in pure strategies. This implies in particular, that there is no policy p with $\Pi^1(p, p_{eq}) > \frac{1}{2}$. Obviously, the converse

[27] A complication arises if one tries to explicitly compute optimal marginal tax rates on the basis of this formula. The reason is that, in general, the welfare weights are endogenous objects which depend themselves on the function y^W. However, under additional assumptions about the utility function Φ, the distribution F and the cost-of-effort-function $\tilde{v}$, a formula for optimal marginal tax rates can be derived which depends only on exogenous parameters, see Diamond (1998).

implication is also true: If there is no policy p with $\Pi^1(p, p_{eq}) > \frac{1}{2}$, then (p_{eq}, p_{eq}) is a pure strategy Nash equilibrium, and hence, a pure strategy equilibrium exists.

Corollary 3 is useful because our previous analysis has singled out the policy p_{eq} as the one-and-only equilibrium candidate. Thus a pure strategy equilibrium exists if and only if it is impossible to defeat this policy.

Without imposing additional assumptions it is difficult to check whether there is a policy that wins a majority against p_{eq}. The reason is that such a policy has to involve a double deviation from p_{eq}. If a politician deviates from p_{eq} only by allocating favors differently than under G_{eq} he will not be able to win. Similarly, if he deviates only by having a different provision rule for publicly provided goods or a different income tax schedule, he will not win. These observations follow from the proof of Theorem 1. Thus, p_{eq} can be defeated only by a policy that has a different allocation of favors, and a different provision rule or income tax schedule, if at all.

In the following, we will look at special cases of our general setup so as to provide conditions for the existence of pure strategy equilibria.[28]

A one-type economy with an indivisible public good. We consider a special case of the model in Section 3: The set Θ is a singleton, so that all individuals have the same type, which we denote by θ. There is a pure public good which comes as an indivisible unit: Either the provision level is 1, or the provision level is 0. If the public good is provided, all individuals realize a utility of $v(\theta, 1) = \theta$. If it is not provided they realize a utility of $v(\theta, 0) = 0$. The per capita cost of providing the public good is given by k. For simplicity, we assume that $\theta > k$, so that surplus-maximization requires to provide the public good, $q^* = 1$.

Proposition 1 *An equilibrium exists if and only if $\theta \geq 2k$.*

The case of an indivisible public good that is either provided or not makes it particularly easy to see that only a double deviation from the equilibrium policy has a chance to defeat it. If politician 2 behaves according to the equilibrium policy and proposes $q = 1$, then politician 1 can defeat him only by proposing $q = 0$. If he proposes $q = 0$, then the probability that he gets a particular voter can be shown to be a convex function of the favors that are offered to that voter. This implies that the best distribution of favors for politician 1 is an extreme one that mixes between an offer of 0 and an offer that attracts the voter with probability 1. As we show in the proof of Proposition 1 the condition $\theta \geq 2k$ holds if and only if such a policy is unable to win a majority against p_{eq}, i.e. against the policy that involves $q = 1$ and a uniform distribution of favors over the interval $[0, 2(e - k)]$.

[28] These simple examples involve a discrete number types. The general treatment of the discrete type case is relegated to the Online-Appendix.

Proposition 1 also shows that the existence of a surplus-maximizing pure strategy equilibrium cannot be taken for granted. If $\theta > k$, then surplus-maximization requires to provide the public good. If at the same time $\theta < 2k$, then it is possible to defeat the surplus-maximizing policy p_{eq} by offering a transfer of $\theta + 2(e - k)$ to more than half of the electorate. Any voter who receives such an offer will prefer the inefficient policy over p_{eq}. By contrast, if the benefit from public-goods provision is sufficiently large, then the fraction of voters that can be attracted in this manner is smaller than $\frac{1}{2}$ which implies that a pure strategy equilibrium exists.[29]

A two-type model of income taxation. We consider a model of income taxation with two types, so that $\Omega = \{\omega_1, \omega_2\}$.

Proposition 2 *Suppose that $\bar{y}(\omega_1) = y^*(\omega_1)$. Then an equilibrium exists.*

Using the definition of marginal income tax rates, see equation (13), the condition $y(\omega_1) \leq y^*(\omega_1)$ can as well be interpreted as the requirement that marginal income tax rates at the bottom have to lie between 0 and 1. From the perspective of the theory of optimal income taxation this is a weak requirement. In the Mirrleesian model, optimal marginal income tax rates lie between 0 and 1, at all income levels and not just at the bottom, see e.g. Hellwig (2007*a*).

The proof of Proposition 2 can be found in Appendix A.3. The condition $\bar{y}(\omega_1) = y^*(\omega_1)$ implies that the vote-share of a politician who runs against the equilibrium candidate p_{eq} is concave in pork-barrel spending. Jensen's inequality then implies that a vote-share maximizing politician refrains from offering different amounts of pork-barrel to different voters. Formally, the best he can do is to choose G as a degenerate distribution. The arguments in the proof of Theorem 2 then imply that his best response is to propose the surplus-maximizing y-function. Absent the condition $y(\omega_1) \leq y^*(\omega_1)$, we can not rule out the possibility that p_{eq} is defeated by a policy that involves inefficiently high output provision for the low-skilled and a distribution of pork that has two mass points so that some voters receive a lot of pork and others do not receive anything.

6.2 Mixed strategy equilibrium

Suppose that there is a policy p' that wins a majority against the equilibrium candidate p_{eq}. Then, by Corollary 3, a pure strategy equilibrium does not exist. The existence of an equilibrium in mixed strategies can be ensured in a straightforward way by a restriction of the policy space. Let P be a finite set of admissible policies that includes p', p_{eq} and

[29] This observation has been made before by Lizzeri and Persico (2001), albeit in the context of a different setup. They assumed that $e = k$, so that the provision of the public good and redistribution via G are dichotomous policy choices. They also provide a characterization of the mixed strategy equilibrium that arises if $\theta < 2k$.

possibly a set of welfare-maximizing policies. With such a finite policy domain, mixed strategy equilibria exist by standard arguments, see Fudenberg and Tirole (1991).

For a game-theoretic analysis of political competition, different interpretations of mixed strategy equilibria have been proposed. For instance, Lizzeri and Persico (2001) interpret a mixed strategy equilibrium as representing the uncertainty about the way in which politicians select among pure strategies. Alternatively, a mixed strategy equilibrium can be interpreted as a pure strategy equilibrium of an extended game in which politicians also choose a distribution of perceptions about what they are going to do if elected. The probability that a politician makes a specific proposal under a mixed strategy can then be identified with the population share of voters who are convinced that this is the policy that he would implement, see Laslier (2000). Here, we do not attempt to provide a characterization of mixed strategy equilibria for the cases in which p_{eq} can be defeated.[30] Rather, we are interested in the efficiency and welfare properties of such equilibria.

Corollary 4 *Suppose that there is a policy p' that defeats the equilibrium candidate p_{eq}. Suppose that the policy domain is a finite set P of admissible policies that includes p', p_{eq} and a subset of P^W. Any mixed strategy equilibrium has the following properties:*

(a) Policies in P^W are played with a probability that is strictly smaller than one.

(b) Surplus-maximizing policies are played with a probability that is strictly smaller than one.

Part (a) of Corollary 4 is an immediate implication of Theorem 2. If one politician played policies that belong to P^W with probability 1, then his opponent could defeat him by playing p_{eq} with probability 1, so that this cannot be an equilibrium. Part (b) is an implication of Theorem 1. This theorem implies that the only candidate for an equilibrium in which surplus-maximizing policies are played with probability one is such that both politicians propose p_{eq}. But then any one politician can ensure a victory by deviating to p', which is, again, incompatible with an equilibrium.

The significance of Corollary 4 is as follows: It shows that whether or not a pure strategy equilibrium exists is not only a technical question. Non-existence of a pure strategy equilibrium can be interpreted as a political failure: In any mixed strategy equilibrium, policies that are not first-best Pareto-efficient will be played with positive probability.

7 Probabilistic Voting

For a model of income taxation, our previous analysis gave rise to the prediction that marginal tax rates are equal to zero at all income levels. This is not descriptive in an

[30] See Lizzeri and Persico (2001) for such an analysis in a complete information environment.

empirical sense, but an implication of our assumption that political competition is "pure": Voters base their decisions only on the policies that are proposed and not on the identities of the policy-makers. Put differently, politicians have no market power. In the following, we will relax this assumption so that each policy-maker has voters who are likely to vote for him even if he does not offer a more attractive policy than the opponent. Thus, there is "monopolistic" rather than "pure" competition.

We adapt a framework that is known as the probabilistic voting model due to Lindbeck and Weibull (1987) to study its implication for a model with pork-barrel spending and income taxation. This yields the following insight: Equilibrium policies can involve distortionary income taxes. Specifically, we derive a formula that characterizes the marginal tax rates that arise in an equilibrium of the probabilistic voting model. It is akin to the formula for welfare-maximizing marginal tax rates by Diamond (1998), see equation (16), except that the terms in Diamond's formula that relate to the social welfare function are replaced by measures of how sensitive voting behavior is to the proposed income tax policies. Thus, this formula enables a new perspective on studies that use a revealed preferences approach to recover the social welfare function that is consistent with an empirically observed tax policy.[31] Through the lens of our model, what these papers identify are political responses which are shaped both by the voters' ideological predispositions and the way in which competing politicians allocate favors in the electorate.

Given two admissible policies $p^1 = (y^1, c^1, G^1)$ and $p^2 = (y^2, c^2, G^2)$, in case of having type ω, individual i votes for politician 1 provided that

$$x^1 - h(\omega, y^1) > \varepsilon_i^2 + x^2 - h(\omega, y^2) \ ,$$

where x^j is the favor offered by politician j,

$$h(\omega, y^j) := \tilde{v}(\underline{\omega}, y^j(\underline{\omega})) + \int_{\underline{\omega}}^{\omega} \tilde{v}_1(s, y^j(s))ds \ , \tag{17}$$

is the information rent that type ω of voter i realizes under the income tax schedule proposed by politician j. Again, we can represent an admissible policy as a pair $p^j = (y^j, G^j)$ with the understanding that the corresponding c-function then follows from the requirement of incentive compatibility.[32] Finally, ε_i^2 is voter i's bias towards politician 2. It is interpreted as the maximal utility loss from accepting politician 2's policy that voter i would tolerate before switching to politician 1. We assume that ε_i^2 is the realization of

[31] See, e.g., Christiansen and Jansen (1978), Blundell, Brewer, Haan and Shephard (2009), Bourguignon and Spadaro (2012), Bargain, Dolls, Neumann, Peichl and Siegloch (2011), Zoutman, Jacobs and Jongen (2012). For an extensive discussion of this approach and the related literature see Lockwood and Weinzierl (2014). Other papers attempt to explain observed tax policies based on an elicitation of preferences for distributive policies (see, e.g., Cowell and Schokkaert, 2001; Fong, 2001; Corneo and Grüner, 2000; Corneo and Grüner, 2002; Devooght and Schokkaert, 2003; Engelmann and Strobel, 2004; Ackert, Martinez-Vazquez and Rider, 2007; Weinzierl, 2014; Kuziemko, Norton, Saez and Stantcheva, 2015; Saez and Stantcheva, 2013; Durante, Putterman and Weele, 2014).

[32] See Section 4 for a discussion and Lemma A.1 for formal details.

a random variable whose distribution may possibly depend on the individual's type ω. Hence, we allow for the possibility that, say, low income people are, ceteris paribus, more likely to vote for party 1 than high income people.[33] The distribution of ε_i^2, conditional on ω, is represented by an atomless *cdf* $B^2(\cdot \mid \omega)$ with density $b^2(\cdot \mid \omega)$. The letter B indicates that we are formalizing a political bias.

Given p^1 and p^2, politician 1's vote share is now given by

$$\Pi^1(p^1, p^2) = \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} E\left[B^2(x^1 - h(\omega, y^1) - \left(x^2 - h(\omega, y^2)\right) \mid \omega)\right] \, dG^2(x^2) \, dG^1(x^1) \, .$$

Politician 1's best response problem is to choose, given G^2 and y^2, G^1 and y^1 so as to maximize this expression subject to the constraints that y^1 is a non-decreasing function and subject to the budget constraint

$$\int_{\mathbb{R}_+} x^1 \, dG^1(x^1) \leq e + S_v(y^1) \, .$$

For ease of exposition, in the following, we focus on a relaxed problem that does not take the monotonicity constraint on y^1 into account.[34] We can then think of politician 1 as choosing G^1 and y^1 so as to maximize the following Lagrangian

$$\mathcal{L} = \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} L(x^1, y^1 \mid x^2, y^2) \, dG^1(x^1) \, dG^2(x^2) \, ,$$

where

$$L(x^1, y^1 \mid x^2, y^2) = E\left[B^2(x^1 - h(\omega, y^1) - \left(x^2 - h(\omega, y^2)\right) \mid \omega)\right] + \lambda \, S_v(y^1) - \lambda \, x^1 \, ,$$

and λ is the multiplier associated with the resource constraint.

Proposition 3 *Consider the solution of politician 1's relaxed best response problem. The marginal income tax rate for an individual with type ω is given by*

$$T'(y^1(\omega)) := 1 - \tilde{v}_2(\omega, y^1(\omega)) = -\frac{1 - F(\omega)}{f(\omega)} \, (1 - \beta^2(\omega)) \, \tilde{v}_{12}(\omega, y^1(\omega)) \, , \tag{18}$$

where

$$\beta^2(\omega) := \frac{1}{\lambda} \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \bar{b}^2(\omega, x^1, y^1 \mid x^2, y^2) \, dG^1(x^1) \, dG^2(x^2) \, , \tag{19}$$

and

$$\bar{b}^2(\omega, x^1, y^1 \mid x^2, y^2) := E\left[b^2(x^1 - h(s, y^1) - \left(x^2 - h(s, y^2)\right) \mid s) \mid s \geq \omega\right] ,$$

[33] Dixit and Londregan (1998) emphasize the importance of allowing for a correlation between political attitudes and individual characteristics such as income.

[34] As is well known, this constraint may in fact be binding so that a solution to the "full" problem gives rise to bunching of various types at the same income level. It is also well-known how the analysis would have to be modified to take account of this possibility. One makes use of Pontryagin's maximum principle and incorporates the additional constraint $y'(\omega) \geq 0$, see Kamien and Schwartz (1991), Laffont (1988), or Hellwig (2007*a*).

is the measure of voters politician 1 can attract by offering more utility to type ω-individuals, for a given tax schedule y^2 of the opponent, and conditional on the offers of pork being equal to x^1 and x^2.

Equation (18) has the same structure as equation (16) which characterizes a welfare-maximizing tax policy. The only difference is that the term $\beta^2(\omega)$ replaces $\Gamma(\omega)$, the welfare weight of individuals with productivity ω and higher relative to population average; $\beta^2(\omega)$ is a measure of how many additional voters of type ω politician 1 can attract by lowering the marginal tax rate for these voters.[35] Ceteris paribus, the larger $\beta^2(w)$ the lower is the marginal tax rate that politician 1 will propose. The formula reveals that an extension of our basic model that allows for political biases is capable of generating marginal income tax rates that are different from zero.

The term $\beta^2(\omega)$ measures the political return for a tax policy that is more attractive to voters with type ω. It is an endogenous object as it depends on the policies p^1 and p^2 that the politicians propose.[36] Still, one may attempt to estimate these political response functions empirically. Given data on marginal tax rates, the distribution of labor productivity, and labor supply elasticities, one can use equation (18) to solve for $\beta^2(\omega)$, for all $\omega \in \Omega$.[37] One can then check whether these values appear plausible in the light of data on voting behavior. Possibly, the probabilistic voting models offers an explanation for the discrepancy between welfare-maximizing and observed marginal tax rates. For instance, Diamond (1998) and Saez (2001) argue that optimal marginal tax rates for top earners exceed those observed in OECD countries. The probabilistic voting model may offer an explanation for why higher top rates are politically infeasible. A related observation is that high values of $\beta^2(w)$ imply negative marginal tax rates, or, equivalently, a subsidization of earnings. These are inconsistent with a welfare-maximizing approach. It has therefore been a challenge for the literature on optimal income taxation to find conditions under which a program such as the Earned Income Tax Credit may be part of an optimal tax and transfer program.[38] As equation (18) reveals, the probabilistic voting model is in principle capable of generating negative marginal tax rates.

[35] **See also Mueller (2003) for a discussion of the welfare implications of probabilistic voting models.**

[36] **We do not attempt to provide a complete characterization of these equilibrium policies. This is not necessary for the present purpose which is to discuss the empirical implications of the probabilistic voting model for income tax schedules. We also doubt that a complete characterization in terms of the primitives of the model is possible unless one imposes additional assumptions on the distributions $\{B(\cdot \mid \omega)\}_{\omega\in\Omega}$ of political biases.**

[37] To see this, note that, with an iso-elastic cost function of the form $\tilde{v}(\omega, y) = \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}$, equation (18) can be rewritten as

$$\frac{T'(y^1(\omega))}{1 - T'(y^1(\omega))} = \frac{1 - F(\omega)}{\omega f(\omega)} \ (1 - \beta^2(\omega)) \left(1 + \frac{1}{\epsilon}\right) ,$$

where ϵ is the elasticity of labor supply with respect to the net wage.

[38] See e.g. Saez (2002), Choné and Laroque (2010), or Jacquet, Lehmann and Van der Linden (2013).

If one estimates $\beta^2(\omega)$, one estimates a political bias which is shaped both by the proposed tax policies and by the allocation of favors in the electorate. Put differently, with the formula in (19) it will generally not be possible to identify separately the impact of pork-barrel spending and political attitudes on equilibrium tax policies. Still, we can provide a measure of the significance of pork-barrel spending for political equilibrium allocations: In the absence of pork-barrel spending, so that G^1 and G^2 are degenerate distributions that assign probability mass 1 to $e + S_v(y^1)$ and $e + S_v(y^2)$, respectively, equation (18) remains valid except that $\beta^2(\omega)$ becomes

$$\bar{\beta}^2(\omega) = \frac{E\left[b^2(e + S_v(y^1) - h(s, y^1) - (e + S_v(y^2) - h(s, y^2)) \mid s) \mid s \geq \omega\right]}{E\left[b^2(e + S_v(y^1) - h(s, y^1) - (e + S_v(y^2) - h(s, y^2)) \mid s) \mid s \geq \underline{\omega}\right]} . \tag{20}$$

Thus, the difference between $\beta^2(\omega)$ and $\bar{\beta}^2(\omega)$ is a measure of how pork-barrel spending affects the willingness of individuals of type ω to vote for party 1. Independent data on the strength of party loyalties among different types of voters might enable a measurement of $\bar{\beta}^2(\omega)$. If combined with data on $\beta^2(\omega)$, one may ultimately obtain a measure of the impact of pork-barrel spending on tax policy.

8 Alternative Modeling Choices

In the following, we provide a discussion of how alternative modeling choices would affect our main results. We argue that Theorems 1 and 2 extend to a model with a discrete set of types, a model with finitely many individuals, or an analysis in which politicians are given the possibility to allocate pork in a type-contingent way. It also extends, with some adjustments, to an analysis that involves more than just two parties. In addition, we explain to what extent our analysis is dependent on the assumption that preferences are quasi-linear. Finally, for the model of income taxation, we discuss how our analysis would change if voters had a demand for insurance against adverse outcomes later in life, or had themselves a concern for welfare.

Discrete type spaces. We provide a detailed discussion of discrete type spaces in part B.1 of the Online-Appendix. We explain why the discrete type specification warrants separate proofs of Theorems 1 and 2, and provide an equilibrium characterization both for the public goods model and the model of income taxation. In addition, for a two-type model of income taxation, we explicitly compute the vote shares that emerge if one politician proposes the Rawlsian welfare-maximum and his competitor use the equilibrium policy p_{eq} that involves a surplus-maximizing policy that is accompanied by pork-barrel spending. This specific example has a pedagogical value: It illustrates in a clear way that a welfare-maximizer is more attractive to "the poor" but will still lose the election, irrespectively of how numerous "the poor" are.

A finite population. Part B.2 of the Online-Appendix contains an extension of Theorem 1 to an economy with finitely many individuals. A finite population gives rise to one additional complication: The cross-section distribution of types is no longer known, but a random quantity. Policies therefore have to deal with a problem of information aggregation. To be specific, consider the case of a pure public good. In a model with a continuum of individuals, the surplus-maximizing quantity

$$\max_{q \in \mathbb{R}_+} \quad E[v(\theta, q)] - k(q)$$

is known *a priori* by the law of large numbers. In a finite economy with N individuals, however, the number of people who value it highly is a random quantity so that a policy maker has to communicate with individuals in order to learn the vector of types and to be able to compute the value of

$$\max_{q \in \mathbb{R}_+} \quad \frac{1}{N} \left(\sum_{i=1}^{N} v(\theta_i, q) - k(q) \right) .$$

In such a setting each individual is pivotal in the sense that her communication has an influence on the optimal public-goods provision level. However, as we show in the Online-Appendix, there is an extension of our main results to an economy with finitely many individuals.

More general preferences. To what extent do our main results extend beyond the case of quasi-linear preferences? The discussion that follows is based on a formal analysis that is relegated to part B.3 of the Online-Appendix. When we relax the assumption that voters have quasi-linear preferences, we can still show that equilibrium policies are Pareto-efficient in the set of policies that are admissible, i.e. physically feasible and incentive-compatible, see Proposition B.3 in the Online-Appendix. This finding is an extension of Theorem 1, and builds on the following intuition: Vote-share maximizing politicians will not leave opportunities to make voters better off on the table since any such opportunity could be exploited to increase one's vote-share. With quasi-linear preferences, however, we could prove a much stronger statement: Equilibrium policies are first-best, i.e. not only Pareto-efficient in the set of admissible policies, but also Pareto-efficient in the much larger set of physically feasible policies.

Also, allowing for more general preferences, and in particular for income effects, complicates the analysis in that we cannot provide a complete political equilibrium characterization in terms of the primitives of the model. With quasi-linear preferences this is possible, see Corollaries 1 and 2.[39]

In the Online-Appendix, we also provide a "natural" condition so that there is no political equilibrium that gives rise to a welfare-maximizing allocation. Specifically, we

[39] Normative approaches share this feature. For instance, optimal marginal tax rates admit a complete characterization in terms of exogenous variables if preferences are quasi-linear, see Diamond (1998), but not otherwise.

show that to every welfare-maximizing policy we can find an alternative policy that is preferred by a majority of voters, see Proposition B.4 in the Online-Appendix. This finding is akin to Theorem 2. With quasi-linear preferences, by contrast, we could prove that there is one specific policy that defeats any welfare-maximizing policy.

Type-dependent pork-barrel spending. Our analysis was based on the assumption that politicians offer pork to individuals with no attempt to condition these offers on the individual's type. Any such conditioning is limited to the mechanism for public goods provision or income taxation. In the following, we argue that this modeling choice is without loss of generality: If we gave politicians the option to allocate pork in a type-dependent way, then, in equilibrium, they would refrain from using that option.

Consider the model of income taxation. If politician j engages in type-dependent pork-barrel spending we can equivalently think of her as offering an income tax schedule that makes use of randomization devices. The private goods consumption of type ω-individuals is then determined by a randomization device that is specific to this type. We may allow for the possibility that the individuals' output requirements are also determined by a type-dependent randomization device.[40] Incentive compatibility then requires that type ω-individuals prefer their outcomes to be generated by "their" randomization devices rather than by the randomization devices that have been programmed for some other type ω'. Now, hypothesize a race between politicians 1 and 2 and suppose that politician 2 proposes the equilibrium policy p_{eq} that we characterized in Corollary 2. As we argued in Section 4, to achieve a maximal vote-share politician 1 then has to propose surplus-maximizing output requirements. Since the individuals' have convex effort costs the surplus cannot be increased by allowing for randomized output requirements. So, even if randomization devices are allowed for, the best response of politician 1 is still to propose the deterministic surplus-maximizing output function. The requirement of incentive compatibility then pins down the expected consumption levels of different types of individuals, with a degree of freedom in the intercept of this consumption schedule. Again politician 1 cannot increase his vote share by means of randomizing these consumption levels. Due to quasi-linearity, individuals care only for their expected consumption levels. Politician 1 can therefore, with no consequence for her vote share, replace any randomized consumption schedule by a deterministic one that offers the expected consumption level with probability 1.

More political parties. Our main results extend to a model with more than two parties: In a symmetric equilibrium, policies are surplus-maximizing. Moreover, any deviation from the equilibrium policy to one that is welfare-maximizing comes with a loss of votes. In part B.4 of the Online-Appendix we provide a sketch of the argument.

[40] For an analysis of the desirability of randomized income taxation from a welfare perspective, see Hellwig (2007*b*).

Again, we combine insights form Myerson (1993) with an analysis of incentive-compatible policies. Myerson has shown that a larger number of parties implies that the equilibrium distribution of pork-barrel changes in particular way: Parties deviate from a uniform distribution of favors and assign more probability mass to small favors and less to large ones. As shown in the Online-Appendix, this change in the distribution of pork does not affect the welfare properties of equilibrium policies.

Social Insurance and Altruism. In our previous analysis, voters have learned their types when voting over mechanisms for public-goods provision or income taxation. This is important for our results. If, at the time of voting, individuals were still awaiting the realization of a preference or skill-shock, they would evaluate policies with an expected utility function that gives rise to a demand for insurance against adverse realizations of the shock. For instance, individual i would evaluate the income tax schedule proposed by politician j according to

$$E[\Phi(x_i^j + c^j(\omega) - \tilde{v}(\omega, y^j(\omega)))],$$

where x_i^j is the amount of pork that politician j has offered.

Assuming that individuals are altruistic or have a concern for the welfare that is generated by redistributive income taxation is an alternative that has a similar implications as the assumption that individuals have a demand for social insurance. Suppose that type ω of individual i evaluates the income tax schedule proposed by politician j according to a convex combination of his own payoff and the welfare that this policy generates,

$$\alpha(x_i^j + c^j(\omega) - \tilde{v}(\omega, y^j(\omega))) + (1-\alpha)\ E[\gamma(\omega)U(\omega \mid p)],$$

where $\alpha \in (0,1)$ is the weight on the own payoff. This modeling choice also gives a role to welfare considerations in the individuals' evaluation of tax policies.

If voters have a concern for welfare, politicians will choose to offer some welfare-improvement relative to a surplus-maximizing outcome. This could be shown formally by an analysis that is akin to our treatment of the probabilistic voting model: Treating the distributions of pork-barrel G^1 and G^2 as exogenously fixed, possibly at their equilibrium levels, we can derive the marginal tax rates that a vote-share maximizing politician would propose and verify that they will typically be different from zero.

The important insight is that a demand for social insurance or an altruistic attitude towards others will lead to an equilibrium tax policy that involves distortionary income taxes. This force is likely to play an important role in reality. Voters may support a redistributive welfare state if they consider the possibility that they may themselves become dependent on the welfare state in the future or if they have a concern for the well-being of those who currently are.

9 Concluding remarks

The core of the analysis in this paper looks at pure competition between vote-share maximizing politicians. The "purity" has two dimensions: First there is no *a priori* restriction on the set of admissible policies, politicians can propose any policy that respects the economy's information structure and the economy's resource constraint. Second, competition is pure in that neither politicians nor voters have ideological biases or partisan motives. Also, there is no incumbency advantage or any other difference in valence.

A main insight is that equilibrium policies are Pareto-efficient in a first best sense, even though voters have private information on preferences over mechanisms for public goods provision or income tax schedules. Our analysis shows that the requirement that admissible policies have to respect not only resource constraints but also informational constraints – or equivalently incentive compatibility constraints – is key for this insight. Indeed, these incentive constraints imply that a preferential treatment of subsets of the electorate becomes more difficult. If the deal for particular voter types becomes too good, then other voters will claim that they are also of the type that is eligible for the preferential treatment. This limits a politician's ability to channel resources from one group of voters to another, and, as a consequence, surplus-maximizing policies emerge in political equilibrium.

By the *Taxation Principle*, see Hammond (1979) and Guesnerie (1995), this finding admits a different interpretation. The incentive compatibility constraints which emerge in a private information environment are equivalent to the implementability constraints which emerge in a decentralized economic system, i.e. a system where individuals make choices subject to constraints that are affected by government policy. To give examples of such policies, think of households that choose labor supply and consumption expenditures subject to a budget constraint which is shaped by an income tax function, or think of households who decide how much publicly provided health-insurance to acquire, given a menu of tariffs. Hence, our main result is relevant for a society in which individuals are free to choose both economically and politically. According to our main result, political equilibria in such a free society, give rise to surplus-maximizing outcomes.

This result is akin to the first welfare theorem which refers to competitive equilibrium allocation, as opposed to political equilibrium allocations. There is, however, no counterpart to the second welfare theorem. Political equilibria do not give rise to welfare-maximizing outcomes, and this may be interpreted as a political failure. For a model of redistributive income taxation, in which welfare is the standard policy objective, our results imply that political equilibria give rise to an undesirable laissez-faire outcome.

References

Acemoglu, D., M. Golosov and A. Tsyvinski. 2008. "Political Economy of Mechanisms." *Econometrica* 76:619–641.

Acemoglu, D., M. Golosov and A. Tsyvinski. 2010. "Dynamic Mirrlees Taxation under Political Economy Constraints." *Review of Economic Studies* 77:841–881.

Ackert, L., J. Martinez-Vazquez and M. Rider. 2007. "Social Preferences and Tax Policy Design: Some Experimental Evidence." *Economic Inquiry* 45(3):487–501.

Al-Najjar, N. 2004. "Aggregation and the Law of Large Numbers in Large Economies." *Games and Economic Behavior* 47:1–35.

Armstrong, M. and J. Vickers. 2001. "Competitive Price Discrimination." *RAND Journal of Economics* 32(4).

Bargain, O., M. Dolls, D. Neumann, A. Peichl and S. Siegloch. 2011. "Tax-Benefit Systems in Europe and the US: Between Equity and Efficiency." *IZA Discussion Papers 5440* .

Baron, D.P. and J.A. Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83:1181–1206.

Barro, R. 1973. "The Control of Politicians: An economic model." *Public Choice* 14:19–42.

Battaglini, M. and S. Coate. 2008. "A Dynamic Theory of Public Spending, Taxation and Debt." *American Economic Review* 98:201–236.

Besley, T. and S. Coate. 1997. "A Model of Representative Democracy." *Quarterly Journal of Economics* 112:85–114.

Besley, T. and S. Coate. 1998. "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis." *American Economic Review* 88:139–156.

Bierbrauer, F.J. 2011. "On the Optimality of Optimal Income Taxation." *Journal of Economic Theory* 146:2105–2116.

Bierbrauer, F.J., C. Brett and J.A. Weymark. 2013. "Strategic nonlinear income tax competition with perfect labor mobility." *Games and Economic Behavior* 82(0):292 – 311.

Bierbrauer, F.J. and P.C. Boyer. 2013. "Political competition and Mirrleesian income taxation: A first pass." *Journal of Public Economics* 103:1–14.

Bierbrauer, F.J. and P.C. Boyer. 2014. "The Pareto-Frontier in a simple Mirrlees model of income taxation." *Annals of Economics and Statistics* 113–114:185–206.

Blundell, R., M. Brewer, P. Haan and A. Shephard. 2009. "Optimal income taxation of lone mothers: an empirical comparison of the UK and Germany." *Economic Journal* 119:F101–F121.

Bourguignon, F. and A. Spadaro. 2012. "Tax-benefit Revealed Social Preferences." *Journal of Economic Inequality* 10(1):75–108.

Brett, C. and J.A. Weymark. 2014. "Citizen Candidates and Voting Over Incentive-Compatible Nonlinear Income Tax Schedules." *CSDI Working Paper: 6-2014* .

Carbonell-Nicolau, O. and E.A. Ok. 2007. "Voting over income taxation." *Journal of Economic Theory* 134:249–286.

Casamatta, G., H. Cremer and P. De Donder. 2010. "Repeated electoral competition over nonlinear income tax schedules." *Social Choice and Welfare* 35:535–574.

Choné, P. and G. Laroque. 2010. "Negative Marginal Tax Rates and Heterogeneity." *American Economic Review* 100(5):pp. 2532–2547.

Christiansen, V. and E. Jansen. 1978. "Implicit Social Preferences in the Norwegian System of Indirect Taxation." *Journal of Public Economics* 10(2):217–245.

Corneo, G. and H.P. Grüner. 2000. "Social Limits to Redistribution." *American Economic Review* 90:1491–1507.

Corneo, G. and H.P. Grüner. 2002. "Individual Preferences for Political Redistribution." *Journal of Public Economics* 83:83–107.

Cowell, F. and E. Schokkaert. 2001. "Risk perceptions and distributional judgments." *European Economic Review* 45(4-6):941–952.

Crutzen, B. and N. Sahuguet. 2009. "Redistributive politics with distortionary taxation." *Journal of Economic Theory* 144:264–279.

d'Aspremont, C. and L. Gérard-Varet. 1979. "Incentives and Incomplete Information." *Journal of Public Economics* 11:25–45.

Devooght, K. and E. Schokkaert. 2003. "Responsibility-sensitive Fair Compensation in Different Cultures." *Social Choice and Welfare* 21:207–242.

Diamond, P.A. 1998. "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." *American Economic Review* 88:83–95.

Dixit, A. and J. Londregan. 1998. "Ideology, Tactics, and Efficiency in Redistributive Politics." *Quarterly Journal of Economics* 113(2):pp. 497–529.

Downs, A. 1957. *An Economic Theory of Democracy.* New York, Harper and Row.

Drazen, A. 2000. *Political Economy in Macroeconomics.* Princeton University Press.

Durante, R., L. Putterman and J. Weele. 2014. "Preferences for redistribution and perception of fairness: An experimental study." *Journal of the European Economic Association* 12(4):1059–1086.

Engelmann, D. and M. Strobel. 2004. "Inequality Aversion, Efficiency and Maximin Preferences in Simple Distribution Experiments." *American Economic Review* 94(4):857–869.

Fahri, E. and I. Werning. 2008. "The Political Economy of Non-Linear Capital Taxation." Mimeo, MIT.

Feldman, M. and C. Gilles. 1985. "An Expository Note on Individual Risk Without Aggregate Uncertainty." *Journal of Economic Theory* 35:26–32.

Ferejohn, J. 1986. "Incumbent Performance and electoral control." *Public Choice* 50:5–25.

Fong, C. 2001. "Social Preferences, Self-interest, and the Demand for Redistribution." *Journal of Public Economics* 82(2):225–246.

Fudenberg, D. and J. Tirole. 1991. *Game Theory.* Cambridge Massachusetts.

Funk, P. and C. Gathmann. 2013. "How do electoral systems affect fiscal policy? Evidence from cantonal parliaments, 1890-2000." *Journal of the European Economic Association* 11:1178–1203.

Gagliarducci, S., T. Nannicini and P. Naticchioni. 2011. "Electoral Rules and Politicians' Behavior: A Micro Test." *American Economic Journal: Economic Policy* 3(3):144–74.

Green, J. and J.J. Laffont. 1977. "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods." *Econometrica* 45:472–487.

Grossman, G. and E. Helpman. 2008. Party Discipline and Pork-Barrel Politics. In *Institutions and Economic Performance*, ed. E. Helpman. Harvard University Press, Cambridge, MA.

Guesnerie, R. 1995. *A Contribution to the Pure Theory of Taxation.* Cambridge University Press.

Güth, W. and M.F. Hellwig. 1986. "The Private Supply of a Public Good." *Journal of Economics* Supplement 5:121–159.

Hammond, P. 1979. "Straightforward Individual Incentive Compatibility in Large Economies." *Review of Economic Studies* 46:263–282.

Hellwig, M.F. 2005. "A Utilitarian Approach to the Provision and Pricing of Excludable Public Goods." *Journal of Public Economics* 89:1981–2003.

Hellwig, M.F. 2007*a*. "A Contribution to the Theory of Optimal Utilitarian Income Taxation." *Journal of Public Economics* 91:1449–1477.

Hellwig, M.F. 2007*b*. "The undesirability of randomized income taxation under decreasing risk aversion." *Journal of Public Economics* 91(34):791 – 816.

Jacquet, L., E. Lehmann and B. Van der Linden. 2013. "Optimal redistributive taxation with both extensive and intensive responses." *Journal of Economic Theory* 148(5):1770 – 1805.

Judd, K. 1985. "The Law of Large Numbers with a Continuum of I.I.D. Random Variables." *Journal of Economic Theory* 35:19–25.

Kamien, M. and N. Schwartz. 1991. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. Elsevier, New York.

Kovenock, D. and B. Roberson. 2009. "Inefficient redistribution and inefficient redistributive politics." *Public Choice* 139:263–272.

Kuziemko, I., M. Norton, E. Saez and S. Stantcheva. 2015. "How Elastic are Preferences for Redistribution? Evidence from Randomized Survey Experiments." *American Economic Review* 105(4):1478–1508.

Laffont, J.-J. 1988. *Fundamentals of Public Economics*. MIT Press.

Laslier, J.-F. 2000. "Interpretation of electoral mixed strategies." *Social Choice and Welfare* 17:283–292.

Laslier, J.-F. and N. Picard. 2002. "Distributive Politics and electoral competition." *Journal of Economic Theory* 103:106–130.

Ledyard, J. and T. Palfrey. 1999. "A Characterization of Interim Efficiency with Public Goods." *Econometrica* 67:435–448.

Lehmann, E., L. Simula and A. Trannoy. 2014. "Tax me if you can! Optimal Nonlinear Income Tax Between Competing Governments." *Quarterly Journal of Economics* 129(4):1995–2030.

Lindbeck, A. and J. Weibull. 1987. "Balanced-budget Redistribution as the Outcome of Political Competition." *Public Choice* 52:273297.

Lizzeri, A. 1999. "Budget Deficits and Redistributive Politics." *Review of Economic Studies* 66:909–928.

Lizzeri, A. and N. Persico. 2001. "The Provision of Public Goods under Alternative Electoral Incentives." *American Economic Review* 91(1):225–239.

Lizzeri, A. and N. Persico. 2004. "Why Did the Elites Extend the Suffrage? Democracy and the Scope of Government, with an Application to Britain's Age of Reform." *Quarterly Journal of Economics* 119(2):707–765.

Lizzeri, A. and N. Persico. 2005. "A Drawback of Electoral Competition." *Journal of the European Economic Association* 3(6):1318–1348.

Lockwood, B. and M. Weinzierl. 2014. "Positive and Normative Judgements Implicit in U.S. Tax Policy." *Working paper, Harvard University* 14-119.

Mailath, G. and A. Postlewaite. 1990. "Asymmetric Bargaining Procedures with Many Agents." *Review of Economic Studies* 57:351–367.

Martimort, D. 2001. "Optimal Taxation and Strategic Budget Deficit under Political Regime Switching." *Review of Economic Studies* 68:573–592.

Martimort, D. 2006. "Multi-Contracting Mechanism Design." *in Advances in Economics and Econometrics. R. Blundell, W. Newey and T. Persson (eds).* 1:56–101.

Mas-Colell, A., M. Whinston and J. Green. 1995. *Microeconomic Theory.* Oxford University Press.

Meltzer, A. and S. Richard. 1981. "A Rational Theory of the Size of Government." *Journal of Political Economy* 89:914–927.

Milgrom, P. and I. Segal. 2002. "Envelope Theorems for Arbitrary Choice Sets." *Econometrica* 70:583–601.

Mirrlees, J. 1971. "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies* 38:175–208.

Mueller, D. 2003. *Public Choice III.* Cambridge University Press.

Mussa, M. and S. Rosen. 1978. "Monopoly and Product Quality." *Journal of Economic Theory* 18:301–317.

Myerson, R. 1993. "Incentives to Cultivate Favored Minorities Under Alternative Electoral Systems." *American Political Science Review* 87(4):856–869.

Osborne, M. and A. Rubinstein. 1994. *A course in Game Theory.* MIT Press, Cambridge, MA.

Osborne, M. and A. Slivinski. 1996. "A Model of Political Competition With Citizen-Candidates." *Quarterly Journal of Economics* 111:65–96.

Persson, T and G. Tabellini. 2000. *Political Economics: Explaining Economic Policy.* Cambridge, MA, MIT Press.

Röell, A.A. 2012. "Voting over Nonlinear Income Tax Schedules." *Working Paper* .

Roberson, B. 2006. "The Colonel Blotto game." *Economic Theory* 29:1–24.

Roberson, B. 2008. "Pork-Barrel Politics, Tagetable Policies, and Fiscal Federalism." *Journal of the European Economic Association* 6(4):819–844.

Roberts, K. 1977. "Voting over Income Tax Schedules." *Journal of Public Economics* 8:329–340.

Rochet, J.-C. and L.A. Stole. 2002. "Nonlinear Pricing with Random Participation." *Review of Economic Studies* 69(1):pp. 277–311.

Roemer, J.E. 2011. "A theory of income taxation where politicians focus upon core and swing voters." *Social Choice and Welfare* 36:383–421.

Saez, E. 2001. "Using Elasticities to Derive Optimal Income Tax Rates." *Review of Economic Studies* 68:205–229.

Saez, E. 2002. "Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses." *Quarterly Journal of Economics* 117(3):pp. 1039–1073.

Saez, E. and S. Stantcheva. 2013. "Generalized Social Marginal Welfare Weights for Optimal Tax Theory." *NBER Working Paper No. 18835* .

Sahuguet, N. and N. Persico. 2006. "Campaign spending regulation in a model of redistributive politics." *Economic Theory* 28:95–124.

Samuelson, P. 1954. "The Pure Theory of Public Expenditure." *Review of Economics and Statistics* 36:387–389.

Sheshinski, E. 1972. "The Optimal Linear Income-tax." *Review of Economic Studies* 39:297–302.

Stiglitz, J. 1982. "Self-Selection and Pareto-Efficient Taxation." *Journal of Public Economics* 17:213–240.

Sun, Y. 2006. "The exact law of large numbers via Fubini extension and characterization of insurable risks." *Journal of Economic Theory* 126:31–69.

Weinzierl, M.C. 2014. "The Promise of Positive Optimal Taxation: Normative Diversity and a Role for Equal Sacrifice." *Journal of Public Economics,* 118:128–142.

Wilson, J.D. 1980. "The Effect of Potential Emigration on the Optimal Linear Income Tax." *Journal of Public Economics* 14:339–53.

Wilson, J.D. 1992. "Optimal Income Taxation and International Personal Mobility." *American Economic Review, Papers and Proceedings* 82(2):pp. 191–196.

Zoutman, F., B. Jacobs and E. Jongen. 2012. "Revealed Social Preferences of Dutch Political Parties." *Tinbergen Institution Discussion Paper* .

A Appendix

A.1 Proofs of Theorems 1 and 2

A.1.1 Preliminaries

Characterization of admissible policies. The following lemma provides a characterization of admissible policies. It is based on standard arguments, in particular on the characterization of incentive compatible policies and their budgetary implications – see, for instance, Mas-Colell, Whinston and Green (1995) and Milgrom and Segal (2002). We therefore state it without proof.

Lemma A.1 *Suppose that q is a continuously differentiable function. Then, a policy $p = (q, c, G)$ is admissible if and only if it satisfies the following constraints:*

(i) Monotonicity: q is a non-decreasing function.

(ii) Utility: for all θ,

$$c(\theta) + v(\theta, q(\theta)) = v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds \ . \tag{21}$$

with

$$E\left[v(\bar{\theta}, q(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q(s))ds\right] = v(\bar{\theta}, q(\bar{\theta})) - E\left[\frac{F(\theta)}{f(\theta)} v_1(\theta, q(\theta))\right] \ . \tag{22}$$

(iii) Resource constraint:

$$\int_0^{\infty} x \ dG(x) \leq e + S_v(q) \ , \tag{23}$$

where

$$S_v(q) := E[v(\theta, q(\theta))] - K(q) - \left(v(\bar{\theta}, q(\bar{\theta})) - E\left[\frac{F(\theta)}{f(\theta)} v_1(\theta, q(\theta))\right]\right) \ . \tag{24}$$

(iv) For each θ, $q(\theta) \in \Lambda(q)$.

Vote-Shares. Upon using equation (21), the probability that any one voter i votes for politician 1 is equal to the probability of the event

$$x_i^2 \leq x_i^1 + v(\bar{\theta}, q^1(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q^1(s))ds - v(\bar{\theta}, q^2(\bar{\theta})) + \int_{\theta}^{\bar{\theta}} v_1(s, q^2(s))ds \ .$$

If the distributions G^1 and G^2 are atomless the vote share of politician 1 can be written as

$$\Pi^1(p^1, p^2) := E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 + h(\theta, q^1) - h(\theta, q^2)\right) dG^1\left(x_i^1\right)\right], \tag{25}$$

where

$$h(\theta, q^j) := v(\bar{\theta}, q^j(\bar{\theta})) - \int_{\theta}^{\bar{\theta}} v_1(s, q^j(s))ds \ . \tag{26}$$

A.1.2 Proof of Theorem 1

The proof of Theorem 1 follows from the sequence of lemmas below.

Lemma A.2 *If the set of equilibria is non-empty, then there exists a symmetric equilibrium.*

Proof The game of political competition is a symmetric constant-sum game. For the properties of such games, see e.g. Osborne and Rubinstein (1994). Now suppose that there is an equilibrium in which politician 1 proposes a policy p_{eq}. For a constant sum-game it holds that p_{eq} is an equilibrium policy if and only if

$$p_{eq} \ \in \text{argmin}_{p^1 \in P} \max_{p^2 \in P} \Pi^2(p^1, p^2) \ ,$$

where P is the set of admissible policies. Since the game is symmetric, if p_{eq} solves this problem, then it is also the case that

$$p_{eq} \ \in \text{argmin}_{p^2 \in P} \max_{p^1 \in P} \Pi^1(p^1, p^2) \ .$$

Hence, (p_{eq}, p_{eq}) is a symmetric Nash-equilibrium. □

Lemma A.3 *Suppose that there is a symmetric equilibrium. Denote by q_{eq} the corresponding provision rule. Then, the equilibrium distribution of lump-sum transfers is a uniform distribution on $[0, 2(e + S_v(q_{eq}))]$.*

Proof If a symmetric equilibrium exists, then it has to be the case that any one politician j choose the distribution G^j so as to maximize his vote share conditional on $q^1 = q^2 = q_{eq}$. Otherwise he could increase his vote share by sticking to the provision rule q_{eq}, but offering a different distribution of lump-sum transfers. Conditional on $q^1 = q^2 = q_{eq}$, the vote share of politician 1 in (28) becomes

$$\Pi^1(p^1, p^2) = \int_{\mathbb{R}_+} G^2\left(x_i^1\right) dG^1\left(x_i^1\right) \ . \tag{27}$$

Given G^2, he chooses G^1 so as to maximize this expression subject to the constraint that $\int_0^\infty x_i^1 \ dG^1(x_i^1) \leq e + S_v(q_{eq})$. Politician 2 solves the analogous problem. Hence, conditional

on $q^1 = q^2 = q_{eq}$, G^1 has to be a best response to G^2 and vice versa. This problem has been analyzed in Theorem 1 of Myerson (1993) who shows that there is a unique pair of functions G^1 and G^2 which satisfy these best response requirements and the symmetry requirement $G^1 = G^2$. Accordingly, G^1 and G^2 both have to be uniform distributions on $[0, 2(e + S_v(q_{eq}))]$. □

Lemma A.4 *Let q_{eq} be a provision rule which is part of a symmetric Nash equilibrium. Let $G^u_{q_{eq}}$ be a uniform distributions on $[0, 2(e + S_v(q_{eq}))]$. Let $G^d_{q_{eq}}$ be a degenerate distribution which puts unit mass on $e + S_v(q_{eq})$. If $p^1 = (q_{eq}, G^u_{q_{eq}})$ is a best response for politician 1 against $p^2 = (q_{eq}, G^u_{q_{eq}})$. Then $\bar{p}^1 = (q_{eq}, G^d_{q_{eq}})$ is also a best response against $p^2 = (q_{eq}, G^u_{q_{eq}})$.*

Proof If $p^1 = (q_{eq}, G^u_{q_{eq}})$ is a best response for politician 1 against $p^2 = (q_{eq}, G^u_{q_{eq}})$, then it yields a vote share of $\frac{1}{2}$ since the game is symmetric. Upon evaluating the expression in (27) under the assumption that $G^2 = G^u$ and $G^1 = G^d_{q_{eq}}$, one verifies that $\bar{p}^1 = (q_{eq}, G^d_{q_{eq}})$ does also generate a vote share of $\frac{1}{2}$. □

Lemma A.5 *If q_{eq} is the provision rule which is part of a symmetric Nash equilibrium, then it is equal to the surplus-maximizing provision rule, i.e. $q_{eq} = q^*$.*

Proof The following observation is an implication of Lemmas A.3 and A.4. If q_{eq} is part of a symmetric Nash equilibrium, then it has to solve the following *constrained best response problem* of politician 1: Choose a provision rule q^1 so as to maximize

$$\Pi^1 = E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 + h(\theta, q^1) - h(\theta, q^2)\right) dG^1\left(x_i^1\right)\right] \tag{28}$$

subject to the following constraints: $G^1 = G^d_{q^1}$, $q^2 = q_{eq}$, and $G^2 = G^u_{q_{eq}}$. Otherwise, politician 1 could improve upon his equilibrium payoff by deviating from q_{eq}, which would be a contradiction to q_{eq} being part of an equilibrium.

We now characterize the solution to the constrained best-response problem. The problem can equivalently be stated as follows: Choose q^1 so as to maximize

$$\Pi^1 = E\left[G^u_{q_{eq}}\left(e + S_v(q^1) + h(\theta, q^1) - h(\theta, q_{eq})\right)\right] \quad . \tag{29}$$

The assumption that admissible provision rules have to be bounded, in combination with the assumption that e is sufficiently large implies that, for all q^1 and for all $\theta \in \Theta$,

$$e + S_v(q^1) + h(\theta, q^1) - h(\theta, q_{eq}) > 0 \ .$$

It also implies that, for all q^1 and for all $\theta \in \Theta$,

$$e + S_v(q^1) + h(\theta, q^1) - h(\theta, q_{eq}) < 2(e + S_v(q_{eq})) \ .$$

Hence, for all θ,

$$G^u_{q_{eq}}\left(e + S_v(q^1) + h(\theta, q^1) - h(\theta, q_{eq})\right) = \frac{e + S_v(q^1) + h(\theta, q^1) - h(\theta, q_{eq})}{2(e + S_v(q_{eq}))} \ ,$$

so that the objective in (29) becomes

$$\begin{aligned}\Pi^1 &= E\left[\frac{e+S_v(q^1)+h(\theta,q^1)-h(\theta,q_{eq})}{2(e+S_v(q_{eq}))}\right] \\ &= \frac{e+S_v(q^1)+E[h(\theta,q^1)]-E[h(\theta,q_{eq})]}{2(e+S_v(q_{eq}))}\ .\end{aligned} \tag{30}$$

By equations (22) and (26), we have that

$$E[h(\theta,q^1)] = v(\bar{\theta},q(\bar{\theta})) - E\left[\frac{F(\theta)}{f(\theta)}v_1(\theta,q(\theta))\right]\ .$$

Upon using equation (24), we find that

$$S_v(q^1)+E[h(\theta,q^1)] = E[v(\theta,q(\theta))]-K(q) = S(q^1)\ .$$

Upon substituting this into (30), we obtain

$$\Pi^1 = \frac{e+S(q^1)-E[h(\theta,q_{eq})]}{2(e+S_v(q_{eq}))}\ . \tag{31}$$

Consequently, politician 1 chooses q^1 so as to maximize $S(q^1)$, which yields $q^1 = q^*$. □

A.1.3 Proof of Theorem 2

Let W be a given welfare function. Upon using the characterization of admissible policies in Lemma A.1, the welfare that is induced by an admissible policy $p = (q, G)$ is given by

$$W(p) = E\left[\gamma(\theta)\int_0^\infty \Phi\left(x+v(\bar{\theta},q(\bar{\theta}))-\int_\theta^{\bar{\theta}} v_1(s,q(s))ds\right)dG(x)\right]\ .$$

Under a welfare-maximizing policy q and G are chosen so as to maximize this expression subject to the constraints that (i) q is a non-decreasing function, (ii) that, for all θ, $q(\theta) \in \Lambda(q)$, and (iii) that the resource constraint holds as an equality, i.e. that $\int_0^\infty x\, dG(x) = e + S_v(q)$. Now, suppose that individuals are risk-averse. Then, the function Φ is strictly concave, so that, by Jensen's inequality,

$$\begin{aligned}&E\left[\gamma(\theta)\textstyle\int_0^\infty \Phi\left(x+v(\bar{\theta},q(\bar{\theta}))-\int_\theta^{\bar{\theta}} v_1(s,q(s))ds\right)dG(x)\right] \\ &< E\left[\gamma(\theta)\Phi\left(e+S_v(q)+v(\bar{\theta},q(\bar{\theta}))-\textstyle\int_\theta^{\bar{\theta}} v_1(s,q(s))ds\right)\right]\ ,\end{aligned}$$

for any non-degenerate distribution G, and any given provision rule q. Hence, a welfare-maximizing policy consists of a degenerate distribution G_q^d which puts unit mass on $e + S_v(q)$ and a provision rule q which maximizes

$$E\left[\gamma(\theta)\Phi\left(e+S_v(q)+v(\bar{\theta},q(\bar{\theta}))-\int_\theta^{\bar{\theta}} v_1(s,q(s))ds\right)\right]$$

subject to the constraints that q is a non-decreasing function and that, for all θ, $q(\theta) \in \Lambda(q)$. We will denote such a policy henceforth by $p^W = (q^W, G^d_{q^W})$. Now consider a policy $p^* = (q^*, G^*)$, where q^* is the surplus-maximizing provision rule and G^* is a uniform distribution on

$[0, 2(e + S_v(q^*))]$. We will now show that $\Pi^1(p^W, p^*) \leq \frac{1}{2}$, with a strict inequality whenever $q^W \neq q^*$. By the arguments in the proof of Lemma A.5,

$$\begin{aligned}
\Pi^1(p^W, p^*) &= \frac{e + S(q^W) - E[h(\theta, q^*)]}{2(e + S_v(q^*))} \\
&\leq \frac{e + S(q^*) - E[h(\theta, q^*)]}{2(e + S_v(q^*))} \\
&= \frac{e + S_v(q^*)}{2(e + S_v(q^*))} \\
&= \frac{1}{2} ,
\end{aligned}$$

where the inequality in the second line is strict whenever $q^W \neq q^*$ and the equality in the third line follows from equations (24), (22) and (26). □

A.2 Proof of Proposition 1

The normalization $c(\overline{\theta}) = 0$ applied to the given setup with a single type implies that $c(\theta) = 0$. An admissible policy is thus a pair $p = (q, G)$, where $q \in \{0, 1\}$ and

$$\int_0^\infty x \, dG(x) \leq e - k \, q \, .$$

If we adapt our equilibrium characterization to this setup, we obtain the equilibrium candidate $p_{eq} = (q_{eq}, G_{eq})$ so that $q_{eq} = q^* = 1$ and G_{eq} is a uniform distribution on $[0, 2(e - k)]$. An equilibrium exists if and only if there is no policy that wins a majority against p_{eq}. It follows from the arguments in the proof Lemma A.3 that there is no such policy that also involves $q = 1$. Hence, we can, without loss of generality, limit attention to policies that involve $q = 0$. Suppose politician 1 chooses such a policy $p^1 = (0, G^1)$, whereas politician 2 chooses $p^2 = p_{eq}$. Politician 1 will then realize a vote share of

$$\Pi^1(p^1, p_{eq}) = \int_0^\infty G_{eq}(x - \theta) dG^1(x) \, .$$

Consider the problem to choose G^1 so as to maximize this expression subject to the constraint that $\int_0^\infty x \, dG^1(x) \leq e$. Since G_{eq} is uniform on $[0, 2(e - k)]$, the optimal G^1 does not involve offers strictly larger than $\theta + 2(e - k)$: A voter who received such an offer would vote for politician 1 with probability 1. The same probability could be generated with an offer that is exactly equal to $\theta + 2(e - k)$ and hence less costly. Hence, the support of G^1 is a subset of $[0, \theta + 2(e - k)]$ and the optimization problem can be rewritten as: Choose G^1 so as to maximize

$$\Pi^1(p^1, p_{eq}) = \int_0^{\theta + 2(e-k)} \max\left\{0, \frac{x - \theta}{2(e - k)}\right\} dG^1(x)$$

subject to $\int_0^{\theta+2(e-k)} x \, dG^1(x) \leq e$. Upon exploiting the convexity of the objective function, one can show that it is optimal to choose G^1 such that only an offer of 0 and an offer of $\theta + 2(e - k)$ are made with positive probability.[41] Denote by $\bar{G}^1$ the probability that politician 1 offers

[41] Here is a sketch of the formal argument: Offers in $]0, \theta[$ will be made with probability zero, because they yield a zero probability of winning the voter but require more resources than the offer of zero.

$\theta + 2(e-k)$. It follows from the resource constraint $\int_0^{\theta+2(e-k)} x \, dG^1(x) = e$ that $\bar{G}^1 = \frac{e}{\theta+2(e-k)}$. This yields a vote share of $\Pi^1(p^1, p_{eq}) = \bar{G}^1 = \frac{e}{\theta+2(e-k)}$. Thus, an equilibrium exists if and only if this vote share is below $\frac{1}{2}$, i.e. if and only if

$$\frac{e}{\theta + 2(e-k)} \leq \frac{1}{2} \quad \Longleftrightarrow \quad \theta \geq 2k \; .$$

□

A.3 Proof of Proposition 2

The proof has four steps:

Step 1. For a model with two types, the equilibrium candidate $p_{eq} = (y_{eq}, c_{eq}, G_{eq})$ looks as follows:[42] The pre tax-incomes are surplus-maximizing, so that, for any $k \in \{1, 2\}$, $y_{eq}(\omega_k) = y^*(\omega_k) = \text{argmax}_y \; y - \tilde{v}(\omega_k, y)$; the consumption function c_{eq} is such that $c_{eq}(\omega_1) = 0$, and $c_{eq}(\omega_2) = \tilde{v}(\omega_2, y^*(\omega_2)) - \tilde{v}(\omega_2, y^*(\omega_1))$; and finally, G_{eq} is a uniform distribution on $[0, 2(e + S_v(y^*))]$, where

$$S_v(y^*) \;\; := \;\; S(y^*) + \tilde{v}(\omega_1, y^*(\omega_1)) + f_2(\tilde{v}(\omega_2, y^*(\omega_1)) - \tilde{v}(\omega_1, y^*(\omega_1))) \; ,$$

and

$$S(y^*) \;\; := \;\; f_1(y^*(\omega_1) - \tilde{v}(\omega_1, y^*(\omega_1))) + f_2(y^*(\omega_2) - \tilde{v}(\omega_2, y^*(\omega_2))) \; .$$

We now check whether this policy can be defeated. Suppose that politician 1 chooses an arbitrary policy $p^1 = (y^1, c^1, G^1)$ with $c^1(\omega_1) = 0$ whereas politician behaves according to the equilibrium candidate p_{eq}. Politician 1 realizes a vote share of

$$\begin{aligned}
\Pi^1(p, p_{eq}) \;\; &= \;\; f_1 \int_0^\infty G_{eq}(x - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1)))dG^1(x) \\
&\quad + f_2 \int_0^\infty G_{eq}(x + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) - (c_{eq}(\omega_2) - \tilde{v}(\omega_2, y^*(\omega_2))))dG^1(x) \\
&= \;\; f_1 \int_0^\infty G_{eq}(x - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1)))dG^1(x) \\
&\quad + f_2 \int_0^\infty G_{eq}(x + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) + \tilde{v}(\omega_2, y^*(\omega_1)))dG^1(x) \; ,
\end{aligned}$$

where the second equality exploits the fact that under p_{eq} downward incentive constraints hold as equalities, so that $c_{eq}(\omega_2) - \tilde{v}(\omega_2, y^*(\omega_2)) = -\tilde{v}(\omega_2, y^*(\omega_1))$.

Hence, the support of G^1 is contained in $0 \cup [\theta, \theta + 2(e-k)]$. Now suppose that offers in an interval $[\underline{x}, \overline{x}] \subset]\theta, \theta+2(e-k)[$ are made with positive probability. One can show that a decrease of the probability of offers in that interval accompanied by a simultaneous increase of the probability that an offer of $\theta + 2(e-k)$ is made – where these changes are such that the budget constraint $\int_0^{\theta+2(e-k)} x \, dG(x) = e$ remains intact – yields an increase of $\Pi^1(p^1, p_{eq})$. This shows that it cannot be optimal to make offers that belong to $]\theta, \theta + 2(e-k)[$.

[42] The Online-Appendix of this paper contains a complete equilibrium characterization for the discrete-type-version of the model.

To be admissible the policy p^1 has to respect the following constraints: Upper bounds on incomes $y^1(\omega_1) \leq \bar{y}(\omega_1)$ and $y^1(\omega_2) \leq \bar{y}(\omega_2)$; incentive compatibility

$$\tilde{v}(w_1, y^1(\omega_2)) - \tilde{v}(w_1, y^1(\omega_1)) \geq c(\omega_2) \geq \tilde{v}(w_2, y^1(\omega_2)) - \tilde{v}(w_2, y^1(\omega_1)) \ , \tag{32}$$

and the resource constraint

$$\int_0^\infty x \, dG^1(x) + f_2 \, c^1(\omega_2) \leq e + f_1 \, y^1(\omega_1) + f_2 \, y^1(\omega_2) \ . \tag{33}$$

Step 2. We now show that, given arbitrary y^1 and c^1 that are part of an admissible policy, the objective $\Pi^1(p^1, p_{eq})$ is a concave function of pork-barrel payments, denoted by x. To this end we verify that, for all $x \geq 0$,

$$x - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1)) \geq 0 \ , \tag{34}$$

and

$$x + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) + \tilde{v}(\omega_2, y^*(\omega_1)) \geq 0 \ . \tag{35}$$

The concavity of Π^1 in x then follows from the fact that G_{eq} is concave over the positive reals. To verify that (34) and (35) hold, note that $y^*(\omega_1) = \bar{y}(\omega_1) = \text{argmax}_{y' \in \mathbb{R}_+} \ y' - \tilde{v}(\omega, y')$ implies in particular that $y^1(\omega_1) \leq y^*(\omega_1)$, for every admissible policy. Hence, $x - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1)) \geq 0$, for all $x \geq 0$. Incentive compatibility, see (32), implies that

$$x + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) + \tilde{v}(\omega_2, y^*(\omega_1)) \geq x - \tilde{v}(w_2, y^1(\omega_1)) + \tilde{v}(\omega_2, y^*(\omega_1)) \geq 0 \ ,$$

where the second inequality follows, once more, from $y^1(\omega_1) \leq y^*(\omega_1)$.

Step 3. Consider the problem to choose $p^1 = (y^1, c^1, G^1)$ so as to maximize $\Pi^1(p^1, p_{eq})$ subject to the requirement that p^1 is admissible. Since, for given y^1 and c^1, Π^1 is concave in x, Jensen's inequality implies that there is a best response where G^1 is a degenerate distribution that puts unit mass on $r^1 := e + f_1 \, y^1(\omega_1) + f_2 \, y^1(\omega_2) - f_2 c^1(\omega_2)$. This yields a vote share of

$$\begin{aligned}
\Pi^1(p^1, p_{eq}) &= f_1 G_{eq}(r^1 - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1))) \\
&\quad + f_2 G_{eq}(r^1 + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) + \tilde{v}(\omega_2, y^*(\omega_1))) \\
&= f_1 \min\left\{1, \frac{r^1 - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1))}{2(e + S_v(y^*))}\right\} \\
&\quad + f_2 \min\left\{1, \frac{r^1 + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) + \tilde{v}(\omega_2, y^*(\omega_1))}{2(e + S_v(y^*))}\right\} \\
&= f_1 \frac{r^1 - \tilde{v}(\omega_1, y^1(\omega_1)) + \tilde{v}(\omega_1, y^*(\omega_1))}{2(e + S_v(y^*))} \\
&\quad + f_2 \frac{r^1 + c^1(\omega_2) - \tilde{v}(\omega_2, y^1(\omega_2)) + \tilde{v}(\omega_2, y^*(\omega_1))}{2(e + S_v(y^*))} \\
&= \frac{e + S(y^1) + \tilde{v}(\omega_1, y^*(\omega_1)) + f_2(\tilde{v}(\omega_2, y^*(\omega_1)) - \tilde{v}(\omega_1, y^*(\omega_1)))}{2(e + S_v(y^*))} \ ,
\end{aligned}$$

where the third equality follows from our assumptions that e is a sufficiently large number and that $y(\omega_1)$ and $y(\omega_2)$ are bounded from above.

Step 4. We can now complete the proof. The argument in Step 3 implies that, for any admissible $p^1 = (y^1, c^1, G^1)$, we have

$$\begin{aligned}
\Pi^1(p^1, p_{eq}) &= \frac{e + S(y^1) + \tilde{v}(\omega_1, y^*(\omega_1)) + f_2(\tilde{v}(\omega_2, y^*(\omega_1)) - \tilde{v}(\omega_1, y^*(\omega_1)))}{2(e + S_v(y^*))} \\
&\leq \frac{e + S(y^*) + \tilde{v}(\omega_1, y^*(\omega_1)) + f_2(\tilde{v}(\omega_2, y^*(\omega_1)) - \tilde{v}(\omega_1, y^*(\omega_1)))}{2(e + S_v(y^*))} \\
&= \frac{e + S_v(y^*)}{2(e + S_v(y^*))} = \frac{1}{2} \, .
\end{aligned}$$

□

A.4 Proof of Proposition 3

In the following we treat G^1 and G^2 as given, possibly at their equilibrium levels and consider politician 1's choice of the function y^1. We use the Gâteau derivative to derive necessary conditions for an optimal choice of y^1. As a first step, in politician 1's objective function, we replace y^1 by $y^1 + \epsilon\kappa$, where ϵ is a scalar and κ is a function that belongs to $\mathcal{L}^2([\underline{\omega}, \overline{\omega}])$. Upon making use of the shorthand

$$a(x^1, y^1, x^2, y^2, \omega) = x^1 - h(\omega, y^1) - \left(x^2 - h(\omega, y^2)\right)$$

this yields

$$\begin{aligned}
\tilde{\Pi}^1 \;:=\; & \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \int_{\underline{\omega}}^{\overline{\omega}} B^2 \left(a(x^1, y^1 + \epsilon\,\kappa, x^2, y^2, \omega) \mid \omega\right) \, dG^2(x^2) \, dG^1(x^1) dF(\omega) \\
& + \lambda \Big\{ \int_{\underline{\omega}}^{\overline{\omega}} \Big[y^1(\omega) + \epsilon\kappa(\omega) - \tilde{v}(\omega, y^1(\omega) + \epsilon\kappa(\omega)) + \Big(\tilde{v}\left(\underline{\omega}, y^1(\underline{\omega}) + \epsilon\kappa(\underline{\omega})\right) \\
& \qquad + \tfrac{1-F(\omega)}{f(\omega)} \tilde{v}_1(\omega, y^1(\omega) + \epsilon\kappa(\omega)) \Big) \Big] \, dF(\omega) - \int_{\mathbb{R}_+} x^1 dG^1(x^1) \Big\} \, .
\end{aligned}$$

The first order condition requires that for all $\kappa \in \mathcal{L}^2([\underline{\omega}, \overline{\omega}])$, $\frac{\partial \tilde{\Pi}^1}{\partial \epsilon} \mid_{\epsilon=0} = 0$. In particular, these first order conditions have to hold for all functions κ with $\kappa(\underline{w}) = 0$. This yields

$$\begin{aligned}
0 \;=\; & \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \int_{\underline{\omega}}^{\overline{\omega}} \Big(- \int_{\underline{\omega}}^{\omega} \tilde{v}_{12}(s, y^1(s)) \kappa(s) ds \Big) b^2 \left(a(x^1, y^1, x^2, y^2, \omega) \mid \omega\right) \, dG^2(x^2) \, dG^1(x^1) dF(\omega) \\
& + \lambda \Big\{ \int_{\underline{\omega}}^{\overline{\omega}} \Big[\kappa(\omega) - \kappa(\omega) \tilde{v}_2(\omega, y^1(\omega)) + \tfrac{1-F(\omega)}{f(\omega)} \tilde{v}_{12}(\omega, y^1(\omega)) \kappa(\omega) \Big] \, dF(\omega) \Big\} \, .
\end{aligned}$$

After an application of Fubini's theorem, this equation can be written as

$$0 \;=\; \int_{\underline{\omega}}^{\overline{\omega}} H(s) \, \kappa(s) \, ds \, .$$

with

$$\begin{aligned}
H(s) \;:=\; & -\tilde{v}_{12}(s, y^1(s)) \int_{\underline{\omega}}^{\omega} \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} b^2 \left(a(x^1, y^1, x^2, y^2, \omega) \mid \omega\right) dG^2(x^2) dG^1(x^1) dF(\omega) \\
& + \lambda \Big[1 - \tilde{v}_2(s, y^1(s)) + \tfrac{1-F(s)}{f(s)} \tilde{v}_{12}(s, y^1(s)) \Big] \, f(s) \, .
\end{aligned}$$

This equation has to hold for any function κ with $\kappa(\underline{\omega}) = 0$. This requires that $H(s) = 0$, for all s, which is in turn equivalent to equation (18). This proves Proposition 3. □

B Online-Appendix

B.1 Discrete type spaces

Why are separate proofs for Theorems 1 and 2 needed if the set of types is discrete? Consider the public goods setup. With a continuum of types, the requirement of incentive-compatibility reduces the dimensionality of the policy space in a very convenient way. The envelope theorem implies that $u'(\theta \mid q, c) = v_1(\theta, q(\theta))$. Hence,

$$\begin{aligned} u(\theta \mid q, c) &= u(\overline{\theta} \mid q, c) - \int_{\theta}^{\overline{\theta}} u'(s \mid q, c) ds \\ &= v(\overline{\theta}, q(\overline{\theta})) - \int_{\theta}^{\overline{\theta}} v_1(s, q(s)) ds \ , \end{aligned}$$

and therefore

$$\begin{aligned} c(\theta) &= u(\theta \mid q, c) - v(\theta, q(\theta)) \\ &= v(\overline{\theta}, q(\overline{\theta})) - \int_{\theta}^{\overline{\theta}} v_1(s, q(s)) ds - v(\theta, q(\theta)) \ . \end{aligned}$$

Consequently, if the provision function q is given, there is no longer a degree of freedom in the choice of the c-function. This simplification is not available if the set of types is discrete. With a discrete set of types, for any given pair (q, G) there are many c-functions with the property that the triple (q, c, G) is incentive compatibility. A separate step in the proof therefore is to show which c-function emerges in equilibrium. As we show below, if equilibrium existence is ensured, then there is an equilibrium in which c_{eq} has the following property: Given (q_{eq}, G_{eq}), it yields the lowest value of $E[c(\theta)]$ in the set of c-functions with the property that (q_{eq}, c, G_{eq}) is incentive-compatible. This implies that all local upward incentive constraints hold as an equality, i.e. for any one $\theta_l < \overline{\theta}$,

$$c_{eq}(\theta_l) + v(\theta_l, q_{eq}(\theta_l)) = c_{eq}(\theta_{l+1}) + v(\theta_l, q_{eq}(\theta_{l+1})) \ .$$

The logic is as follows. By incentive compatibility, higher types consume more of the publicly provided good and less of the private good. Therefore, minimal private goods consumption is achieved if any one individual's private goods consumption is chosen as close as possible to the consumption level of the next higher type.

Equilibrium policies if the set of types is discrete. The proofs of Theorems 1 and 2 for a model with a discrete set of types yield the following equilibrium characterization for our model of public goods provision.

Corollary B.1 *Suppose that the set of types is discrete. If an equilibrium exists, then the unique symmetric equilibrium $p_{eq} = (q_{eq}, c_{eq}, G_{eq})$ is such that:*

(a) The provision rule is surplus-maximizing $q_{eq} = q^*$.

(b) Private goods consumption is such that that $c_{eq}(\overline{\theta}) = 0$ and for any $\theta_l < \overline{\theta}$,

$$\begin{aligned} c_{eq}(\theta_l) &= v(\overline{\theta}, q^*(\overline{\theta})) - v(\theta_l, q^*(\theta_l)) \\ &\quad - \sum_{k=l}^{n-1} \{v(\theta_{k+1}, q^*(\theta_{k+1})) - v(\theta_k, q^*(\theta_{k+1}))\} \ , \end{aligned} \tag{36}$$

(c) G_{eq} *is uniform on* $[0, 2(e + S_v(q^*))]$, *where*

$$\begin{aligned} S_v(q^*) \quad := \quad & S(q^*) - \Big\{ v(\overline{\theta}, q^*(\overline{\theta})) \\ & - \sum_{k=1}^{n-1} f(\theta_k) \tfrac{F(\theta_k)}{f(\theta_k)} \{ v(\theta_{k+1}, q^*(\theta_{k+1})) - v(\theta_k, q^*(\theta_{k+1})) \} \Big\} \ . \end{aligned} \tag{37}$$

The adaptation of Corollary B.1 to the model of income taxation yields the following corollary.

Corollary B.2 *Suppose that* $\Omega = \{\omega_1, \ldots, \omega_n\}$ *with* $\omega_1 = \underline{\omega}$ *and* $\omega_n = \overline{\omega}$. *If the set of equilibria is non-empty, then the unique symmetric equilibrium* $p_{eq} = (y_{eq}, c_{eq}, G_{eq})$ *is such that:*

(a) Before-tax-incomes are surplus-maximizing $y_{eq} = y^*$.

(b) Private goods consumption is such that $c(\underline{\omega}) = 0$ *and for any* $\omega_k > \underline{\omega}$,

$$\begin{aligned} c_{eq}(\omega_k) \quad = \quad & \tilde{v}(\omega_k, y^*(\omega_k)) - \tilde{v}(\underline{\omega}, y^*(\underline{\omega})) \\ & - \textstyle\sum_{l=1}^{k-1} \{ \tilde{v}(\omega_{l+1}, y^*(\omega_l)) - \tilde{v}(\omega_l, y^*(\omega_l)) \} \ . \end{aligned} \tag{38}$$

(c) G_{eq} *is uniform on* $[0, 2(e + S_v(y^*))]$, *where*

$$\begin{aligned} S_v(y^*) \quad := \quad & S(y^*) + \Big(\tilde{v}(\underline{\omega}, y^*(\underline{\omega})) \\ & + \textstyle\sum_{l=1}^{n-1} f(\omega_l) \frac{1-F(\omega_l)}{f(\omega_l)} \{ \tilde{v}(\omega_{l+1}, y^*(\omega_l)) - \tilde{v}(\omega_l, y^*(\omega_l)) \} \Big) \ . \end{aligned} \tag{39}$$

Characterization of admissible policies if the set of types is discrete. The following lemma is the analogue to Lemma A.1. It characterizes admissible pairs consisting of a provision rule q and a lottery G.

Lemma B.1 *Consider a pair* (q, G). *There is a c-function so that* $p = (q, c, G)$ *is an admissible policy if and only if the following properties hold:*

(i) Monotonicity: q *is a non-decreasing function.*

(ii) Resource constraint:

$$\int_0^\infty x \, dG(x) \leq e + S_v(q) \ , \tag{40}$$

where

$$\begin{aligned} S_v(q) \quad := \quad & E[v(\theta, q(\theta))] - K(q) - \Big(v(\theta_n, q(\theta_n)) \\ & - \sum_{k=1}^{n-1} f(\theta_k) \tfrac{F(\theta_k)}{f(\theta_k)} \{ v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1})) \} \Big) \ . \end{aligned} \tag{41}$$

(iii) For each θ, $q(\theta) \in \Lambda(q)$.

We do not provide a formal proof of Lemma B.1 because it relies on known arguments, see e.g. Mussa and Rosen (1978) or Hellwig (2007*a*). However, we provide a sketch of the main arguments. The monotonicity requirement in (i) is an implication of incentive compatibility and the constraint in (iii) is a physical constraint. Now suppose that we have a pair (q, G) in which q satisfies these requirements. The question then is whether we can find a function $c : \theta \to c(\theta)$ so that the triple $p = (q, c, G)$ is admissible. To this end we study an auxiliary problem: Fix q and G, and then choose c so as to minimize $E[c(\theta)]$ subject to incentive constraints. Denote the solution to this problem by c_{min}. The proof is based on the following insight: If $\int_0^\infty x \; dG(x) + E[c_{min}(\theta)] \leq e - K(q)$, then $p = (q, c_{min}, G)$ is an admissible policy. If, by contrast, $\int_0^\infty x \; dG(x) + E[c_{min}(\theta)] > e - K(q)$, then it is impossible to find a c-function so that (q, G) is part of an admissible policy: If it is impossible to meet the resource constraint with the "cheapest" consumption function, then it is not possible to meet it at all.

The formula in (41) follows from $\int_0^\infty x \; dG(x) + E[c_{min}(\theta)] \leq e - K(q)$, in combination with a characterization of c_{min}. At a solution to the auxiliary problem all local upward incentive constraints are binding, i.e., for all $\theta_l < \theta_n = \overline{\theta}$,

$$c_{min}(\theta_l) + v(\theta_l, q(\theta_l)) = c_{min}(\theta_{l+1}) + v(\theta_l, q(\theta_{l+1})) \; .$$

This insight, in combination with the normalization that $c(\theta_m) = 0$ makes it possible to solve for all consumption levels as a function of the provision rule q: For all $\theta_l < \theta_n$,

$$c_{min}(\theta_l) = v(\theta_n, q(\theta_n)) - v(\theta_l, q(\theta_l)) - \sum_{k=l}^{n-1} \{v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1}))\} \; , \tag{42}$$

and hence

$$\begin{aligned} E[c_{min}(\theta)] \quad = \quad & -E[v(\theta, q(\theta))] + v(\theta_n, q(\theta_n)) \\ & - \sum_{k=1}^{n-1} f(\theta_k) \tfrac{F(\theta_k)}{f(\theta_k)} \{v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1}))\} \; . \end{aligned} \tag{43}$$

Upon substituting this expression into $\int_0^\infty x \; dG(x) + E[c_{min}(\theta)] \leq e - K(q)$, we obtain the inequality in (41).

In the following, we represent an admissible policy $p^j = (q^j, c^j, G^j)$ for politician j as a triple (q^j, Δ^j, G^j) in which $\Delta^j : \theta \to \Delta^j(\theta)$ is defined such that

$$c^j(\theta_l) = c^j_{min}(\theta_l) + \Delta^j(\theta_l) \; .$$

Lemma B.2 *Let $p^j = (q^j, \Delta^j, G^j)$ be an admissible policy. Then, for all θ, $\Delta^j(\theta) \geq 0$.*

Proof The local upward incentive-compatibility constraints, for all $\theta_l < \theta_n$,

$$c(\theta_l) + v(\theta_l, q(\theta_l)) \geq c(\theta_{l+1}) + v(\theta_l, q(\theta_{l+1}))$$

imply, in particular that, for any $\theta_l < \theta_n$,

$$c(\theta_l) \quad \geq \quad c(\theta_n) + v(\theta_n, q(\theta_n)) - v(\theta_l, q(\theta_l)) - \sum_{k=l}^{n-1} \{v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1}))\} \; ,$$

$$= v(\theta_n, q(\theta_n)) - v(\theta_l, q(\theta_l)) - \sum_{k=l}^{n-1} \{v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1}))\}$$
$$= c_{min}(\theta_l) \ ,$$

where the equality in the second line follows from $c(\theta_n) = c_{min}(\theta_n) = 0$ and the equality in the third line follows from (42). □

Upon using (42), the utility that an individual of type θ_l derives form (q^j, c^j, Δ^j) can be written as

$$\begin{aligned} c^j(\theta_l) + v(\theta_l, q^j(\theta_l)) &= \Delta^j(\theta_l) + c^j_{min}(\theta_l) + v(\theta_l, q^j(\theta_l)) \\ &= \Delta^j(\theta_l) + v(\theta_n, q^j(\theta_n)) \\ &\quad - \sum_{k=l}^{n-1} \{v(\theta_{k+1}, q^j(\theta_{k+1})) - v(\theta_k, q^j(\theta_{k+1}))\} \ . \end{aligned}$$

Thus, taking account of pork-barrel spending, the probability that any one voter i votes for politician 1 is equal to the probability of the event

$$x_i^2 \leq x_i^1 + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q^1) + \Delta^2(\theta)) \ .$$

where

$$h(\theta_l, q^j) := v(\theta_n, q(\theta_n)) - \sum_{k=l}^{n-1} \{v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1}))\} \ , \tag{44}$$

for $\theta_l < \theta_n$, and

$$h(\theta_n, q^j) := v(\theta_n, q(\theta_n)) \ . \tag{45}$$

If the distributions G^1 and G^2 are atomless the vote share of politician 1 can be written as

$$\Pi^1(p^1, p^2) := E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 + h(\theta, q^1) - h(\theta, q^2) + \Delta^1(\theta) - \Delta^2(\theta)\right) dG^1\left(x_i^1\right)\right] \ . \tag{46}$$

For later reference we note that

$$E[h(\theta, q^j)] = v(\theta_n, q(\theta_n)) - \sum_{k=1}^{n-1} f(\theta_k) \frac{F(\theta_k)}{f(\theta_k)} \{v(\theta_{k+1}, q(\theta_{k+1})) - v(\theta_k, q(\theta_{k+1}))\} \ . \tag{47}$$

and

$$\begin{aligned} E[c^j(\theta)] &= E[c^j_{min}(\theta)] + E[\Delta^j(\theta)] \\ &= -E[v(\theta, q^j(\theta))] + E[h(\theta, q^j)] + E[\Delta^j(\theta)] \ , \end{aligned} \tag{48}$$

so that a policy $p^j = (q^j, \Delta^j, G^j)$ is resource feasible if and only if

$$\int_0^\infty x \ dG(x) \leq e + S_v(q) - E[\Delta^j(\theta)] \ . \tag{49}$$

B.1.1 Proof of Theorem 1 if the set of types is discrete

The proof of Theorem 1 follows from the sequence of lemmas below.

Lemma B.3 *If the set of equilibria is non-empty, then there exists a symmetric equilibrium.*

Proof See the proof of Lemma A.2. □

Lemma B.4 *Suppose that there is a symmetric equilibrium. Denote by* (q_{eq}, Δ_{eq}) *the corresponding provision rules. Then, the equilibrium distribution of favors is a uniform distribution on* $[0, 2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)])]$.

Proof If a symmetric equilibrium exists, then it has to be the case that any one politician j choose the distribution G^j so as to maximize his vote share conditional on $q^1 = q^2 = q_{eq}$ and $\Delta^1 = \Delta^2 = \Delta_{eq}$. Otherwise he could increase his vote share by sticking to (q_{eq}, Δ_{eq}), but offering a different distribution of pork-barrel. Conditional on $q^1 = q^2 = q_{eq}$, and $\Delta^1 = \Delta^2 = \Delta_{eq}$ the vote share of politician 1 in (51) becomes

$$\Pi^1(p^1, p^2) = \int_{\mathbb{R}_+} G^2\left(x_i^1\right) dG^1\left(x_i^1\right) \ . \tag{50}$$

Given G^2, he chooses G^1 so as to maximize this expression subject to the constraint that $\int_0^\infty x_i^1 \ dG^1(x_i^1) \leq e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)]$. Politician 2 solves the analogous problem. Hence, conditional on $q^1 = q^2 = q_{eq}$, and $\Delta^1 = \Delta^2 = \Delta_{eq}$, G^1 has to be a best response to G^2 and vice versa. This problem has been analyzed in Myerson (1993), who shows that there is a unique pair of functions G^1 and G^2 which satisfy these best response requirements and the symmetry requirement $G^1 = G^2$. Accordingly, G^1 and G^2 both have to be uniform distributions on $[0, 2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)])]$. □

Lemma B.5 *Let* (q_{eq}, Δ_{eq}) *be a part of a symmetric Nash equilibrium. Let* $G^u_{q_{eq}}$ *be a uniform distributions on* $[0, 2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)])]$. *Let* $G^d_{q_{eq}}$ *be a degenerate distribution which puts unit mass on* $e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)]$. *If* $p^1 = (q_{eq}, \Delta_{eq}, G^u_{q_{eq}})$ *is a best response for politician 1 against* $p^2 = (q_{eq}, \Delta_{eq}, G^u_{q_{eq}})$. *Then* $\bar{p}^1 = (q_{eq}, \Delta_{eq}, G^d_{q_{eq}})$ *is also a best response against* $p^2 = (q_{eq}, \Delta_{eq}, G^u_{q_{eq}})$.

Proof If $p^1 = (q_{eq}, \Delta_{eq}, G^u_{q_{eq}})$ is a best response for politician 1 against $p^2 = (q_{eq}, \Delta_{eq}, G^u_{q_{eq}})$, then it yields a vote share of $\frac{1}{2}$ since the game is symmetric. Upon evaluating the expression in (50) under the assumption that $G^2 = G^u_{q_{eq}}$ and $G^1 = G^d_{q_{eq}}$, one verifies that $\bar{p}^1 = (q_{eq}, \Delta_{eq}, G^d_{q_{eq}})$ does also generate a vote share of $\frac{1}{2}$. □

Lemma B.6 *If* q_{eq} *is part of a symmetric Nash equilibrium, then* $q_{eq} = q^*$.

Proof The following observation is an implication of Lemmas B.4 and B.5. If q_{eq} is part of a symmetric Nash equilibrium, then it has to solve the following constrained best-response

problem of politician 1: Choose a provision rule q^1 so as to maximize

$$\Pi^1 = E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q^2) + \Delta^2(\theta))\right) dG^1\left(x_i^1\right)\right] \tag{51}$$

subject to the following constraints: $G^1 = G^d_{q_{eq}}$, $q^2 = q_{eq}$, $\Delta^1 = \Delta^2 = \Delta_{eq}$, and $G^2 = G^u_{q_{eq}}$, where $G^u_{q_{eq}}$ and $G^d_{q_{eq}}$ are as defined in Lemma B.5. Otherwise, politician 1 could improve upon his equilibrium payoff by deviating from q_{eq}, which would be a contradiction to q_{eq} being part of an equilibrium.

We now characterize the solution to the constrained best-response problem. The problem can equivalently be stated as follows: Choose q^1 so as to maximize

$$\Pi^1 = E\left[G^u_{q_{eq}}\left(e + S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q_{eq}) + \Delta_{eq}(\theta))\right)\right] \ . \tag{52}$$

Admissible provision rules and hence q^1 and q_{eq} are bounded. The resource constraint also implies that the functions Δ_{eq} and Δ^1 have to be bounded. Hence, the assumption that e is sufficiently large implies that, for all (q^1, Δ^1) and for all $\theta \in \Theta$,

$$e + S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q_{eq}) - \Delta_{eq}(\theta)) \geq 0 \ .$$

It also implies that, for all q^1 and for all $\theta \in \Theta$,

$$e + S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q_{eq}) - \Delta_{eq}(\theta)) \leq 2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)]) \ .$$

Hence, for all θ,

$$\begin{aligned} &G^u_{q_{eq}}\left(e + S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q_{eq}) - \Delta_{eq}(\theta))\right) \\ &\quad = \frac{e + S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q_{eq}) - \Delta_{eq}(\theta))}{2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)])} \ , \end{aligned}$$

so that the objective in (52) becomes

$$\Pi^1 = E\left[\frac{e + S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta) - (h(\theta, q_{eq}) - \Delta_{eq}(\theta))}{2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)])}\right] \ . \tag{53}$$

By equations (47) and (41),

$$E\left[S_v(q^1) - E[\Delta^1(\theta)] + h(\theta, q^1) + \Delta^1(\theta)\right] = E[v(\theta, q(\theta))] - K(q) = S(q^1) \ .$$

Upon substituting this into (53), we obtain

$$\Pi^1 = \frac{e + S(q^1) - E[h(\theta, q_{eq}) - \Delta_{eq}(\theta)]}{2(e + S_v(q_{eq}) - E[\Delta_{eq}(\theta)])} \ . \tag{54}$$

Consequently, politician 1 chooses q^1 so as to maximize the surplus $S(q^1)$, which yields $q^1 = q^*$. □

Lemma B.7 *If Δ_{eq} is part of a symmetric Nash equilibrium, then $\Delta_{eq}(\theta) = 0$ for all θ.*

Proof Suppose that $q^1 = q^2 = q^* = q_{eq}$ and that G^2 is uniform on $[0, 2(e+S_v(q_{eq})-E[\Delta^2(\theta)])]$.

We first show that if politician 2 chooses $\Delta^2(\theta) = 0$, for all θ, then Δ^1 with $\Delta^1(\theta) = 0$, for all θ, is a best response for politician 1. Politician 1's vote share is

$$\begin{aligned}
\Pi^1 &= E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 + \Delta^1(\theta)\right) dG^1\left(x_i^1\right)\right] \\
&\leq E\left[\frac{e + S_v(q_{eq}) - E[\Delta^1(\theta)] + \Delta^1(\theta)}{2(e + S_v(q_{eq})}\right] \\
&= \frac{e + S_v(q_{eq})}{2(e + S_v(q_{eq})} = \frac{1}{2}\,.
\end{aligned}$$

Moreover, Π^1 equals $\frac{1}{2}$, if $\Delta^1(\theta) = 0$, for all θ and G^1 is uniform on $[0, 2(e + S_v(q_{eq}))]$.

Now suppose that politician 2 chooses Δ^2 so that $\Delta^2(\theta) > 0$ for some θ. We show that politician has a best response that yields a vote share strictly larger than $\frac{1}{2}$, which is incompatible with a Nash equilibrium. Consider a policy p^1 that involves $q^1 = q_{eq}$, $\Delta^1(\theta) = 0$ for all θ, and a distribution G^1 so that only an offer of 0 and an offer of $2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])$ are made with positive probability. We denote by $\bar{G}^1$ the probability that politician 1 offers $2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])$. Since the policy p^1 has to be resource-feasible

$$\bar{G}^1 = \frac{e + S_v(q_{eq})}{2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])}\,. \tag{55}$$

The vote share of politician 1 is given by

$$\begin{aligned}
\Pi^1 &= E\left[\int_{\mathbb{R}_+} G^2\left(x_i^1 - \Delta^2(\theta)\right) dG^1\left(x_i^1\right)\right] \\
&= \bar{G}^1 \frac{2\left(e + S_v(q_{eq}) - E[\Delta^2(\theta)]\right) - E[\Delta^2(\theta)]}{2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])}\,,
\end{aligned}$$

Now $\Pi^1 > \frac{1}{2}$ if

$$\bar{G}^1 \frac{2\left(e + S_v(q_{eq}) - E[\Delta^2(\theta)]\right) - E[\Delta^2(\theta)]}{2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])} > \frac{1}{2} \Leftrightarrow$$

$$\frac{e + S_v(q_{eq})}{2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])} \frac{2\left(e + S_v(q_{eq}) - E[\Delta^2(\theta)]\right) - E[\Delta^2(\theta)]}{2(e + S_v(q_{eq}) - E[\Delta^2(\theta)])} > \frac{1}{2} \Leftrightarrow$$

$$e + S_v(q_{eq}) - 2E[\Delta^2(\theta)] > 0.$$

This last inequality is always true when e is large since $\Delta^2(\theta)$ is bounded.

B.1.2 Proof of Theorem 2 if the set of types is discrete

Let W be a given welfare function. Upon using the characterization of admissible policies in Lemma A.1, the welfare that is induced by an admissible policy $p = (q, G)$ is given by

$$W(p) = E\left[\gamma(\theta) \int_0^\infty \Phi\left(x + h(\theta, q) + \Delta(\theta)\right) dG(x)\right]\,.$$

Under a welfare-maximizing policy q, Δ and G are chosen so as to maximize this expression subject to the constraints that (i) q is a non-decreasing function, (ii) that, for all θ, $q(\theta) \in \Lambda(q)$, and finally, that the resource constraint holds as an equality, i.e. that $\int_0^\infty x\, dG(x) = e + S_v(q) -$

$E[\Delta(\theta)]$. Now, suppose that individuals are risk averse. Then the function Φ is strictly concave, so that, by Jensen's inequality,

$$\begin{aligned}&E\left[\gamma(\theta)\int_0^\infty \Phi\left(x+h(\theta,q)+\Delta(\theta)\right)dG(x)\right]\\&< E\left[\gamma(\theta)\Phi\left(e+S_v(q)-E[\Delta(\theta)]+x+h(\theta,q)+\Delta(\theta)\right)\right]\ ,\end{aligned}$$

for any non-degenerate distribution G, and any given pair (q,Δ). Hence, a welfare-maximizing policy consists of a degenerate distribution G_q^d which puts unit mass on $e+S_v(q)-E[\Delta(\theta)]$ and a pair (q,Δ) which maximizes

$$E\left[\gamma(\theta)\Phi\left(e+S_v(q)-E[\Delta(\theta)]+h(\theta,q)+\Delta(\theta)\right)\right]$$

subject to the constraints of incentive compatibility, that q is a non-decreasing function and that, for all θ, $q(\theta)\in\Lambda(q)$. We will denote such a policy henceforth by $p^W=(q^W,\Delta^W,G_{q^W}^d)$. Now consider a policy $p^*=(q^*,\Delta^*,G^*)$, where q^* is the surplus-maximizing provision rule and G^* is a uniform distribution on $[0,2(e+S_v(q^*))]$ and $\Delta^*(\theta)=0$, for all θ. We will now show that $\Pi^1(p^W,p^*)\leq\frac{1}{2}$, with a strict inequality whenever $q^W\neq q^*$. By the arguments in the proof of Lemma B.6,

$$\begin{aligned}\Pi^1(p^W,p^*) &= \frac{e+S(q^W)-E[h(\theta,q^*)]}{2(e+S_v(q^*))}\\ &\leq \frac{e+S(q^*)-E[h(\theta,q^*)]}{2(e+S_v(q^*))}\\ &= \frac{e+S_v(q^*)}{2(e+S_v(q^*))}\\ &= \frac{1}{2}\ ,\end{aligned}$$

where the inequality in the second line is strict whenever $q^W\neq q^*$ and the equality in the third line follows from (47) and (41). □

B.1.3 A two-type model of income taxation

A central theme of our analysis is that pork-barrel spending has striking implications for an analysis of political competition. To illustrate this point we consider a simple model of non-linear income taxation with two-types of individuals, the less productive and the highly productive. We also assume that the less productive are the bigger group. In the absence of pork-barrel spending, vote-share maximizing politicians will propose the income tax schedule that is most attractive to the bigger group. The assumption that the less productive are more numerous then implies that, in a political equilibrium with two competing parties, both propose the income tax schedule that maximizes a Rawlsian welfare function (see for a proof Bierbrauer and Boyer, 2013). In the following, we illustrate the general insight of Theorem 2 in this context: We show explicitly that the Rawlsian policy is defeated by a surplus-maximizing policy that involves pork-barrel spending.

The set of possible types is taken to be the same for all individuals and given by $\Omega=\{\omega_L,\omega_H\}$, with $\omega_L<\omega_H$. Without a possibility of pork-barrel spending, a policy or allocation $p=(c_L,\Delta c_H,y_L,y_H)$ consists of an after-tax income c_L for low-skilled agents, a number Δc_H

which gives the extra after-tax-income that any one individual receives in case of being high-skilled, a level of pre-tax-income for a low-skilled individual y_L, and a level of pre-tax income for a high-skilled individual y_H. Consequently, the utility of individual i is given by $c_L - \tilde{v}(\omega_L, y_L)$ if $\omega_i = \omega_L$, and given by $c_L + \Delta c_H - \tilde{v}(\omega_H, y_H)$ if $\omega_i = \omega_H$.

A policy has to satisfy physical constraints. First, consumption levels have to be non-negative, so that $c_L \geq 0$, and $c_L + \Delta c_H \geq 0$. Second, $y_L \in [0, \bar{y}_L]$ and $y_L \in [0, \bar{y}_H]$, where $\bar{y}_L$ and $\bar{y}_H$ are maximal levels of income that low-skilled and high-skilled individuals are capable of generating. We assume that

$$\bar{y}_L \geq y_L^s := \text{argmax}_y \; y - \tilde{v}(\omega_L, y) \; , \quad \text{and} \quad \bar{y}_H \geq y_H^s := \text{argmax}_y \; y - \tilde{v}(\omega_H, y) \; ,$$

and refer to y_L^s and y_H^s as the first-best, or surplus-maximizing levels of pre-tax-income, with the surplus being defined as

$$S(y_L, y_H) := f_L(y_L - \tilde{v}(\omega_L, y_L)) + f_H(y_H - \tilde{v}(\omega_H, y_H)) \; .$$

Finally, the economy's resource constraint has to be satisfied

$$c_L + f_H \Delta c_H \leq e + f_L y_L + f_H y_H \; .$$

Individuals have private information on their types, so that, in addition to the physical constraints above, the following incentive compatibility constraints have to be satisfied,

$$-\tilde{v}(\omega_L, y_L) \geq \Delta c_H - \tilde{v}(\omega_L, y_H) \; , \quad \text{and} \quad \Delta c_H - \tilde{v}(\omega_H, y_H) \geq -\tilde{v}(\omega_H, y_L) \; .$$

The first inequality ensures that, in case of being low-skilled, an individual prefers to generate the pre-tax-income level y_L over generating y_H and being rewarded with additional after-tax income of Δc_H. Likewise, the second inequality ensures that high-type individuals are sufficiently motivated by Δc_H to generate the pre-tax-income level y_H.[43]

The following lemma characterizes the ideal policy for the low-skilled individuals. We omit a proof, because the derivation uses standard arguments.[44] We denote by $p^R = (c_L^R, \Delta c_H^R, y_L^R, y_H^R)$ the policy that maximizes $c_L - \tilde{v}(\omega_L, y_L)$ subject to physical and incentive constraints.[45]

Lemma B.8 *The policy p^R has the following properties:*

(i) The high-skilled have a binding incentive constraint: $\Delta c_H^R = \tilde{v}(w_H, y_H^R) - \tilde{v}(w_H, y_L^R)$.

(ii) The high-skilled have surplus-maximizing pre-tax incomes incomes: $y_H^R = y_H^s$.

[43] In Section 5 we explain why incentive compatibility is equivalent to the decentralizability of a policy by means of an income tax schedule.

[44] See, for instance, Stiglitz (1982) or Hellwig (2007*a*).

[45] The incentive compatibility constraints imply that high-skilled individuals always realize more utility than the low-skilled individuals:

$$\begin{aligned} c_L^R + \Delta c_H^R - \tilde{v}(w_H, y_H^R) &\geq c_L^R - \tilde{v}(w_H, y_L^R) \\ &> c_L^R - \tilde{v}(w_L, y_L^R) \; . \end{aligned}$$

Hence, a planner who focusses on the worst-off individuals will choose the policy p^R.

(iii) The low-skilled have pre-tax incomes which are distorted downwards relative to the surplus-maximizing one: $y_L^R < y_L^s$.

(iv) The low-skilled individuals' after-tax incomes are given by $c_L^R = e + S_v(y_L^R, y_H^s)$ *, where*

$$S_v(y_L^R, y_H^s) := S(y_L^R, y_H^s) + f_L \tilde{v}(w_L, y_L^R) + f_H \tilde{v}(w_H, y_L^R)$$

is the virtual surplus that is generated by y_L^R *and* y_H^s.

The *virtual* surplus accounts for the information rents that the high-skilled individuals receive. It is obtained by plugging the expression for the high skilled individuals' additional after-tax-income $\Delta c_H^R = \tilde{v}(w_H, y_H^R) - \tilde{v}(w_H, y_L^R)$ into the economy's resource constraint. These information rents reduce the surplus that can be allocated to the low-skilled individuals.[46]

Pork-barrel spending. We now introduce policies that involve a distribution of favors in the electorate. Thus, a politician proposes an income tax schedule $(c_L, \Delta c_H, y_L, y_H)$ to voters, and, in addition, he chooses a cross-section distribution G of individual-specific transfers in the population. The transfers are assumed to be specific to individuals and not related to their productive abilities. Consequently, in case of receiving a draw x from the lottery G, the utility of individual i is given by $x + c_L - \tilde{v}(\omega_L, y_L)$ if $\omega_i = \omega_L$, and given by $x + c_L + \Delta c_H - \tilde{v}(\omega_H, y_H)$ if $\omega_i = \omega_H$.

It is convenient to think of $x + c_L$ as the random consumption level of a voter i in case of being low-skilled. We can thus represent a policy that involves pork-barrel spending as a collection $p = (G, \Delta c_H, y_L, y_H)$ that consists of a probability distribution G which determines any one individual's consumption level $x + c_L$ in the event that the individual is low-skilled, a number Δc_H which gives the extra after-tax-income that any one individual receives in case of being high-skilled, a level of pre-tax-income for a low-skilled individual y_L, and a level of pre-tax income for a high-skilled individual y_H.

We now define a policy which is an adaptation of the equilibrium policy in Theorems 1 and 2 below to the given setup. It involves the use of differentiated lump-sum transfers, surplus-maximizing levels of pre-tax-incomes both for high-skilled and for low-skilled individuals as well as a binding incentive constraint for the high-skilled.[47]

Definition B.1 *The policy* $p^s = (G^s, \Delta c_H^s, y_L^s, y_H^s)$ *specifies surplus-maximizing levels of pre-tax income for both types; in addition it has the following properties:*

(i) The high-skilled have a binding incentive constraint: $\Delta c_H^s = \tilde{v}(w_H, y_H^s) - \tilde{v}(w_H, y_L^s)$.

[46] To see this, we compute the utility that low-skilled individuals realize under p^R. It is given by

$$\begin{aligned} \bar{c}_L^R - \tilde{v}(w_L, y_L^R) &= e + S_v(y_L^R, y_H^s) - \tilde{v}(w_L, y_L^R) \\ &= e + S(y_L^R, y_H^s) - f_H(\tilde{v}(w_L, y_L^R) - \tilde{v}(w_H, y_L^R)) \\ &< e + S(y_L^R, y_H^s) \ . \end{aligned}$$

[47] In an income tax model with two types, the set of Pareto-efficient allocations has three segments (see Bierbrauer and Boyer, 2014): (i) A segment in which the incentive constraint for the low-skilled individuals binds, (ii) a segment in which both incentive constraints are slack and before-tax-incomes are surplus-maximizing, and (iii) a segment in which the incentive constraint for the high-skilled individuals binds. Here we look at the allocation which marks the boundary between segments (ii) and (iii).

(ii) A low-skilled individuals' consumption level $x + c_L$ is drawn from a uniform distribution with support $[0, 2(e + S_v(y_L^s, y_H^s))]$, where

$$S_v(y_L^s, y_H^s) := S(y_L^s, y_H^s) + f_L \tilde{v}(w_L, y_L^s) + f_H \tilde{v}(w_H, y_L^s) \ .$$

is the virtual surplus that is generated by y_L^s and y_H^s.

Proposition B.1 *The surplus-maximizing policy p^s wins a majority against the Rawlsian policy p^R.*

Proof Denote by π_L^s the percentage of low-skilled individuals who prefer p^s over p^R. Analogously, we define π_H^s. We seek to show that $f_L\ \pi_L^s + f_H\ \pi_H^s > \frac{1}{2}$. Note that

$$\begin{aligned}
\pi_L^s &= 1 - G^s(c_L^R + \tilde{v}(w_L, y_L^s) - \tilde{v}(w_L, y_L^R)) \\
&= 1 - \frac{e + S_v(y_L^R, y_H^s) + \tilde{v}(w_L, y_L^s) - \tilde{v}(w_L, y_L^R)}{2(e + S_v(y_L^s, y_H^s))} \\
&= \frac{1}{2} + \frac{S_v(y_L^s, y_H^s) - \big(S_v(y_L^R, y_H^s) + \tilde{v}(w_L, y_L^s) - \tilde{v}(w_L, y_L^R)\big)}{2(e + S_v(y_L^s, y_H^s))} \ .
\end{aligned}$$

Analogously, upon using that $\Delta c_H^R = \tilde{v}(w_H, y_H^R) - \tilde{v}(w_H, y_L^R)$ and that $\Delta c_H^s = \tilde{v}(w_H, y_H^s) - \tilde{v}(w_H, y_L^s)$, we derive

$$\begin{aligned}
\pi_H^s &= 1 - G^s(\bar{c}_L^R + \tilde{v}(w_H, y_L^s) - \tilde{v}(w_H, y_L^R)) \\
&= 1 - \frac{e + S_v(y_L^R, y_H^s) + \tilde{v}(w_H, y_L^s) - \tilde{v}(w_H, y_L^R)}{2(e + S_v(y_L^s, y_H^s))} \\
&= \frac{1}{2} + \frac{S_v(y_L^s, y_H^s) - \big(S_v(y_L^R, y_H^s) + \tilde{v}(w_H, y_L^s) - \tilde{v}(w_H, y_L^R)\big)}{2(e + S_v(y_L^s, y_H^s))} \ .
\end{aligned}$$

Consequently, $f_L\ \pi_L^s + f_H\ \pi_H^s > \frac{1}{2}$ holds provided that

$$S_v(y_L^s, y_H^s) - S_v(y_L^R, y_H^s) > f_L\left(\tilde{v}(w_L, y_L^s) - \tilde{v}(w_L, y_L^R)\right) + f_H\left(\tilde{v}(w_H, y_L^s) - \tilde{v}(w_H, y_L^R)\right) \ ,$$

or, equivalently, if $S(y_L^s, y_H^s) > S(y_L^R, y_H^s)$. The latter inequality holds because y_L^R is distorted downwards relative to the surplus-maximizing income level y_L^s. □

Proposition B.1 shows that a policy that targets the majority of low-skilled workers will not win a majority against the surplus-maximizing policy p^s. Incentive constraints make targeted transfers to the low-skilled prohibitively costly. An inspection of the formulas in the proof reveals that a politician who proposes the policy p^s and runs against the Rawlsian policy p^R, will get a higher percentage of the high-skilled votes, $\pi_H^s > \pi_L^s$. Still, the total effect is a vote share larger than $\frac{1}{2}$. Put differently, a purely partisan policy will be defeated by one that tries to appeal to all voter types, albeit to varying degrees.

B.2 The main result in a large, but finite economy

There are N individuals. The set of individuals is denoted by $I = \{1, \ldots, N\}$. Individual i has a set of types $\Theta_i = [\underline{\theta}, \overline{\theta}]$, which is taken to be the same for all individuals (for brevity, we do not

spell out the analysis for a discrete set of types). If individual i has type θ_i, consumes c_i units of a private good and q_i units of a publicly provided good, she realizes utility of $u_i = c_i + v(\theta_i, q_i)$. The types of different individuals are taken to be the realization of *iid* random variables with cumulative distribution function F and density f. In the following we write $\theta = (\theta_1, \ldots, \theta_N)$ for a generic vector of types. Occasionally, we also write $\theta = (\theta_i, \theta_{-i})$. This is an abuse of notation. In the body of the text, θ referred to a typical realization of the random variable θ_i, as opposed to the vector of types.

Policies. A policy consists of (i) a cross-section distribution G of pork-barrel spending, (ii) for each individual i, a function $q_i : \theta \to q_i(\theta)$ that specifies i's consumption of the publicly provided good as a function of the vector of preferences, (iii) for each individual i, a function $c_i : \theta \to c_i(\theta)$ that specifies i's private goods consumption as a function of the vector of preferences. Again, we adopt the normalization that $c_i(\overline{\theta}) = 0$. We write $q(\theta) = (q_1(\theta), \ldots, q_N(\theta))$ for the collection of all individual consumption levels.

Admissible policies. Admissible policies have to be incentive-compatible and resource-feasible. We introduce some notation so as to present the incentive compatibility constraints in a concise way. Let

$$C_i(\hat{\theta}_i) := E_{\theta_{-i}}\Big[c_i(\theta_i, \theta_{-i})\Big]$$

be the expected private goods consumption of individual i in case of communicating type $\hat{\theta}_i$ under a direct mechanism. Let

$$V_i(\theta_i, \hat{\theta}_i) := E_{\theta_{-i}}\Big[v(\theta_i, q_i(\hat{\theta}_i, \theta_{-i}))\Big]$$

be i's expected utility from the publicly provided good in case of having true type θ_i and communicating type $\hat{\theta}_i$. We denote by

$$\mathbf{U}_i(\theta_i, x) = x + U_i(\theta_i) := x + C_i(\theta_i) + V_i(\theta_i, \theta_i)$$

the utility that type θ_i of individual i realizes in a truth-telling equilibrium, provided that he receives a transfer equal to x. Incentive compatibility requires that for all i, θ_i and $\hat{\theta}_i$,

$$U_i(\theta_i) \geq C_i(\hat{\theta}_i) + V_i(\theta_i, \hat{\theta}_i) \ .$$

We require that budget balance holds in expectation so that

$$\int_0^\infty x \, dG(x) + \frac{1}{N} E_\theta \left[\sum_{i=1}^N c_i(\theta)\right] \leq e + \frac{1}{N} E\left[K(q(\theta))\right] \ .$$

A further constraints is that $q_i(\theta) \in \Lambda(q(\theta))$, for all i, and for all θ_i. The budget constraint holds in expectation, and not in an ex-post sense. This can be justified if the number N of individuals is large. In this case, one can appeal to a law of large numbers so that budget balance in expectation is approximately the same as ex-post budget balance. This argument is formally spelled out in Bierbrauer (2011).

The following lemma is an adaptation of Lemma A.1 to the given setup with a finite number of individuals. Again, we omit a proof.

Lemma B.9 *Suppose that all q_i are continuously differentiable functions. Then, a policy $p = (q, c, G)$ is admissible if and only if it satisfies the following constraints:*

(i) Monotonicity: for any θ_i, the function $V(\theta_i, \cdot)$ is non-decreasing.

(ii) Utility: for all θ,

$$U_i(\theta_i) = V_i(\bar{\theta}, \bar{\theta}) - \int_{\theta}^{\bar{\theta}} V_{i1}(s, s) ds \ . \tag{56}$$

(iii) Resource constraint:

$$\int_0^{\infty} x \ dG(x) \leq e + s_v(q) \ , \tag{57}$$

where

$$S_v(q) \ := \ S(q) - \tfrac{1}{N} \left(\textstyle\sum_{i=1}^N V_i(\bar{\theta}, \bar{\theta}) - E_\theta \left[\sum_{i=1}^N \frac{F(\theta_i)}{f(\theta_i)} v_{i1}(\theta_i, q(\theta)) \right] \right) \ , \tag{58}$$

and

$$S(q) \ := \ \tfrac{1}{N} E_\theta \left[\textstyle\sum_{i=1}^N v(\theta_i, q_i(\theta)) \ - \ K(q(\theta)) \right] . \tag{59}$$

(iv) For each θ, $q(\theta) \in \Lambda(q(\theta))$.

Political competition. We assume that two vote share-maximizing politicians propose an admissible policy. Voters evaluate these policies at the ex interim stage, i.e. after having learned their types. They vote for the policy that generates more utility. Theorems 1 and 2 remain valid in the finite economy version of our model, and we refrain from providing separate proofs. The following proposition adapts the equilibrium characterization to the given setup.

Proposition B.2 *If the set of equilibria is non-empty, then the unique symmetric equilibrium $p_{eq} = (q_{eq}, c_{eq}, G_{eq})$ is such that:*

(a) The provision rule is surplus-maximizing $q_{eq} = q^*$.

(b) Private goods consumption is such that for any $\theta_i \in \Theta$,

$$C^{equ}(\theta_i) = V_i^{equ}(\bar{\theta}, \bar{\theta}) - V_i^{equ}(\theta_i, \theta_i) - \int_{\theta}^{\bar{\theta}} V_{i1}^{equ}(s, s) ds \ , \tag{60}$$

(c) G_{eq} is uniform on $[0, 2(e + S_v(q^*))]$, where

$$S_v(q^*) := S(q^*) - \frac{1}{N} \left(\sum_{i=1}^N V_i(\bar{\theta}, \bar{\theta}) - E_\theta \left[\sum_{i=1}^N \frac{F(\theta_i)}{f(\theta_i)} v_{i1}(\theta_i, q^*(\theta)) \right] \right) \ . \tag{61}$$

Proposition B.2 is very similar to Corollary 1. However, in a finite economy, the interpretation of the statement *q^* maximizes the (non-virtual) surplus $S(q)$* is different. Here, this means that, for every vector of preferences $\theta \in \prod_{i=1}^{N} \Theta_i$, $q(\theta) = (q_1(\theta), \ldots, q_N(\theta))$ is chosen so as to maximize the ex-post-surplus

$$\sum_{i=1}^{N} v(\theta_i, q_i(\theta)) - K(q(\theta)) \ ,$$

i.e. we have a separate optimality condition for each state of the economy, or, for each constellation of preferences. In the continuum economy in the body of the text this was not the case. There, the cross-section distribution of preferences was taken to be given by F and hence not to involve genuine randomness. Put differently, there was only one state of the economy for which an outcome had to be specified.

B.3 More general preferences

We cast our discussion in the context of the Mirrleesian model of income taxation. We model the policy space in the following way: As before, politicians choose a distribution of lump-sum transfers or pork in the population. We denote by G^j the distribution of lump-sum transfers that are offered by politician $j \in \{1, 2\}$. In addition, politician j chooses, for every lump-sum transfer x, an accompanying incentive-compatible mechanism that is represented by the functions $y^j : \Omega \times \mathbb{R} \to \mathbb{R}_+$ and $c^j : \Omega \times \mathbb{R} \to \mathbb{R}_+$. Thus, $y^j(\omega, x)$ is the output requirement for an individual with type ω, conditional on receiving a lump-sum transfer equal to x. The private goods consumption of a type ω individual with a lump-sum transfer x equals $x + c^j(\omega, x)$.

The presence of income effects implies that we can no longer specify incentive compatible mechanisms without having to worry about the lump-sum transfers that individuals receive. Now, each transfer comes with its own set of incentive-compatible mechanism. Let $\mathcal{I}(x)$ be the set of incentive compatible mechanisms conditional on a lump-sum transfer equal to x. Formally, a pair of functions $c(\cdot, x) : \Omega \to \mathbb{R}_+$ and $y(\cdot, x) : \Omega \to \mathbb{R}_+$ belongs to $\mathcal{I}(x)$ if and only if, for all ω and ω',

$$u(x + c(\omega, x), y(\omega, x), \omega) \geq u(x + c(\omega', x), y(\omega', x), \omega) \ ,$$

where u is the function that describes the individual's preferences over private goods consumption and output requirements. We assume that it increases in the first argument, $u_1 > 0$, decreases in the second, $u_2 < 0$, and satisfies the single crossing property: For all $(\tilde{y}, \tilde{c}, x) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}$, and all $(\omega', \omega) \in \Omega^2$, $\omega' > \omega$ implies that

$$-\frac{u_2(x + \tilde{c}, \tilde{y}, \omega')}{u_1(x + \tilde{c}, \tilde{y}, \omega')} \leq -\frac{u_2(x + \tilde{c}, \tilde{y}, \omega)}{u_1(x + \tilde{c}, \tilde{y}, \omega)} \ .$$

A pure strategy for politician j consists of the three functions y^j, c^j, and G^j. It is physically feasible if it satisfies non-negativity constraints so that, for all x and ω,

$$x + c^j(\omega, x) \geq 0 \quad \text{and} \quad y^j(\omega, x) \geq 0 \ ,$$

and a budget constraint

$$\int_{\mathbb{R}} x \, dG^j(x) \leq \int_{\mathbb{R}} E[y^j(\omega, x) - c^j(\omega, x)] \, dG^j(x) \ ,$$

where the expectation operator E is with respect to the individuals' productivity parameter ω. A pure strategy is said to be admissible if it satisfies these constraints and if, in addition, the functions $c^j(\cdot, x)$ and $y^j(\cdot, x)$ belong to $\mathcal{I}(x)$, for all $x \in \mathbb{R}$. The set of admissible policies is henceforth denoted by P.

To ensure equilibrium existence we may have to allow for mixed strategies. A pure strategy for any politician is a distribution over lump-sum transfers and incentive-compatible mechanisms so that, conditional on a particular transfer x being drawn, one of the mechanisms in $\mathcal{I}(x)$ is played with probability 1. Mixed strategies allow, in addition, for a randomization over the set $\mathcal{I}(x)$. We represent a mixed strategy for politician j by two distributions G^j and $H^j(\cdot \mid x)$, where G^j is, as before, the distribution over lump-sum transfers and $H^j(\cdot \mid x)$ is a distribution over the incentive-compatible mechanisms in $\mathcal{I}(x)$. For a mixed strategy, physical feasibility requires that

$$\int_{\mathbb{R}} x \, dG^j(x) \leq \int_{\mathbb{R}} \int_{\mathcal{I}(x)} E[y^j(\omega, x) - c^j(\omega, x)] \, dH^j(c^j, y^j \mid x) \, dG^j(x) \ ,$$

and that, for all x and ω, $x + c^j(\omega, x) \geq 0$ and $y^j(\omega, x) \geq 0$, $H^j(\cdot \mid x)$-almost surely. We denote the set of all such mixed strategies by P^m.

Let $p^2 = (G^2, H^2) \in P^m$ be a mixed strategy that is played by politician 2. Under a pure strategy $p^1 = (y^1, c^1, G^1) \in P$, Politician 1's vote share $\Pi^1(p^1, p^2)$ is then equal to

$$\int_{\mathbb{R}} E \left[\int_{\mathbb{R}} \int_{\mathcal{I}(x)} \mathbf{1}\{u(x^1 + c^1(\omega, x), y^1(\omega, x), \omega) \geq V^2(\omega, x^2)\} \, dH^2(c^2, y^2 \mid x^2) \, dG^2(x^2) \right] \, dG^1(x^1) \ ,$$

where $\mathbf{1}\{\cdot\}$ is the indicator function, and $V^2(\omega, x^2) := u(x^2 + c^2(\omega, x^2), y^2(\omega, x^2), \omega)$ is the utility that is realized by an individual with type ω if politician 2 is elected. From the perspective of politician 1, $V^2(\omega, x^2)$ is a random variable with a distribution $\bar{G}^2(\cdot \mid \omega)$ that is a function of G^2 and H^2. We can therefore write politician 1's vote share equivalently as

$$\int_{\mathbb{R}} E \left[\bar{G}^2(u(x^1 + c^1(\omega, x), y^1(\omega, x), \omega) \mid \omega) \right] \, dG^1(x^1) \ .$$

Politician 1's best response problem is to choose the functions y^1, c^1, and G^1 so as to maximize this expression subject to the constraint that $(y^1, c^1, G^1) \in P$.

For a given level of x^1, this objective is akin to a welfare function with weights that are generated by the functions $\bar{G}^2(\cdot \mid \omega)$. Most importantly, politician 1's vote share is a non-decreasing function of the utility levels $u(x^1 + c^1(\omega, x), y^1(\omega, x), \omega)$. This implies that a mechanism (y^1, c^1) in $\mathcal{I}(x^1)$ will be played with positive probability only if there is no other mechanism $(\bar{y}^1, \bar{c}^1) \in \mathcal{I}(x^1)$ that has the same budgetary implications, $E[y^1(\omega, x^1) - c^1(\omega, x^1)] = E[\bar{y}^1(\omega, x^1) - \bar{c}^1(\omega, x^1)]$, and Pareto-dominates it so that, for all ω,

$$u(x^1 + \bar{c}^1(\omega, x), \bar{y}^1(\omega, x), \omega) \geq u(x^1 + c^1(\omega, x), y^1(\omega, x), \omega) \ ,$$

with a strict inequality for some $\omega \in \Omega$. Consequently, any best response (y^1, c^1, G^1) by politician 1 is Pareto-efficient: Given the distribution of lump-sum transfers G^1, it is impossible to find a Pareto-superior admissible policy. It is also impossible to find an alternative specification of G^1 that generates a Pareto-improvement. This would require to have (weakly) higher lump-sum transfers to all voters and would be incompatible with the budget constraint.

A symmetric argument implies that all best responses of politician 2 are Pareto-efficient. Moreover, in any pure or mixed strategy equilibrium, the two politicians play only strategies that are best responses. These observations prove the following proposition which extends Theorem 1 to an environment in which preferences need not be quasi-linear.

Proposition B.3 *Let (p^1, p^2) be a Nash equilibrium in pure or mixed strategies. All policies that arise with positive probability in an equilibrium are Pareto-efficient in the set of admissible policies.*

We now turn to the question whether a welfare-maximizing policy can be reached as the outcome of political competition. The welfare that is generated by an admissible policy $p = (y, c, G) \in P$ is given by

$$\int_{\mathbb{R}} E\left[\gamma(\omega)\ \Phi(u(x + c(\omega, x)), y(\omega, x), \omega)\right]\ dG(x)\ ,$$

where Φ is a concave function that captures the individuals' risk attitudes.

For brevity of exposition, we impose the following assumption, which will imply that welfare-maximizing policies do not involve pork-barrel spending.

Assumption B.1 *Let*

$$W(r) \quad := \quad max_{c,y\in\mathcal{I}(0)}\ E\left[\gamma(\omega)\ \Phi(u(c(\omega, 0), y(\omega, 0), \omega))\right] \quad s.t. \qquad E[y(\omega, 0) - c(\omega, 0)] = r\ .$$

We assume that $W'(r) < 0$ and that $W''(r) \leq 0$.

Assumption B.1 looks at a problem of welfare-maximization in the absence of lump-sum transfers, but with a given revenue requirement of r. The maximal level of welfare that can be generated depends on the revenue requirement and is denoted by $W(r)$. The marginal cost of public funds are equal to the loss in welfare that comes with an increase of the revenue requirement r. They are given by $-W'(r)$. Assumption B.1 postulates that these marginal costs of public funds are positive and a non-decreasing function of the revenue requirement, $-W''(r) \geq 0$.

Lemma B.10 *Suppose that Assumption B.1 holds. Under every-welfare maximizing policy,*

$$E\left[\gamma(\omega)\ \Phi(u(x + c(\omega, x)), y(\omega, x), \omega)\right]$$

is a degenerate random variable that takes the value $W(0)$ with probability 1.

Proof We approach the problem of welfare-maximization as follows: We first view the distribution G as predetermined and solve for the welfare-maximizing functions c and y, given G. We will then discuss the choice of G. This auxiliary problem can be decomposed in two steps. The first step is to assign to each realization of the lump-sum transfer x, a revenue requirement $r(x)$ and then to choose c and y optimally, given x and $r(x)$. The second step then involves a welfare-maximizing choice of the revenue requirements subject to the constraint that

$$\int_{\mathbb{R}} x\ dG(x) = \int_{\mathbb{R}} r(x)\ dG(x).$$

Step 1. Consider the problem of choosing $c(\cdot, x), y(\cdot, x) \in \mathcal{I}(x)$ so as to maximize

$$E\left[\gamma(\omega)\ \Phi(u(x + c(\omega, x)), y(\omega, x), \omega)\right]$$

subject to non-negativity constraints and the constraint that $E[y(\omega, x) - c(\omega, x)] = r(x)$. Upon defining, for every ω, $c'(\omega, 0) := x + c(\omega, x)$ and $y'(\omega, 0) := y(\omega, 0)$, we can equivalently state this problem as follows: Choose $c'(\cdot, 0), y'(\cdot, 0) \in \mathcal{I}(0)$ so as to maximize

$$E\left[\gamma(\omega)\ \Phi(u(c'(\omega, 0)), y'(\omega, 0), \omega)\right]$$

subject to the constraint that

$$E[y'(\omega, 0) - c'(\omega, 0)] = r(x) - x\ .$$

This leads to a welfare level that equals $W(r(x) - x)$.

Step 2. We now consider the problem of choosing the function $r : x \mapsto r(x)$ so as to maximize

$$\int_{\mathbb{R}} W(r(x) - x)\ dG(x) \quad \text{s.t.} \quad \int_{\mathbb{R}} x\ dG(x) = \int_{\mathbb{R}} r(x)\ .$$

The first-order condition of this problem stipulates that the marginal costs of public funds should be equalized across all possible realizations of the lump-sum transfer x. Formally, there is a (non-negative) number λ, so that for all x,

$$W'(r(x) - x) = -\lambda\ .$$

By Assumption B.1, this implies that there is a number λ' so that, for all x, $r(x) - x = \lambda'$. Upon substituting this into $\int_{\mathbb{R}} r(x) - x\ dG(x) = 0$, it follows that $\lambda' = 0$, and hence that, for all x, $r(x) - x = 0$. Thus, whatever G looks like,

$$\int_{\mathbb{R}} W(r(x) - x)\ dG(x) = \int_{\mathbb{R}} W(0)\ dG(x) = W(0)\ .$$

This observation implies in particular, that there is no optimal choice of G. If G is a non-degenerate distribution, a welfare-maximizer will adjust revenue requirements so that the optimal policy is as if G was assigning mass one to a lump-sum transfer of 0 and a revenue requirement of 0. □

Lemma B.10 implies that any welfare-maximizing policy is such that c and y belong to $\mathcal{I}(0)$ with probability 1. Thus, a policy which solves

$$\max_{c,y \in \mathcal{A}}\ E\left[\gamma(\omega)\ \Phi(u(c(\omega, 0)), y(\omega, 0), \omega)\right]$$

is a welfare-maximizing policy, where $\mathcal{A}$ is the set of policies that belong to $\mathcal{I}(0)$ and which satisfy non-negativity constraints and the budget constraint $E[y(\omega, 0) - c(\omega, 0)] \geq 0$.

The observation that any welfare-maximizing policy belongs to the set $\mathcal{A}$ is key to derive the following proposition, which is a weakened version of Theorem 2.

Proposition B.4 *Suppose that the set $\mathcal{A}$ does not contain a Condorcet winner. Then, to any welfare-maximizing policy p^W, there is a policy $p \in \mathcal{A}$, so that $\Pi^1(p^W, p) < \frac{1}{2}$.*

If the set $\mathcal{A}$ does not contain a Condorcet winner this means that to any policy p' in A there is a policy $p \in \mathcal{A}$, so that $\Pi^1(p^W, p) < \frac{1}{2}$. Trivially, this property then also holds for any welfare-maximizing policy in $\mathcal{A}$. We not provide a separate proof that clarifies the conditions under which there is no Condorcet winner in $\mathcal{A}$. This would lead us astray. It is well-known that, with a multi-dimensional policy space, there is typically no hope of finding a Condorcet winner, see e.g. the discussion in Persson and Tabellini (2000). Applied to the problem of non-linear income taxation, the logic is as follows: Suppose that there are at least three types of individuals in the population and that no type has a mass that exceeds $\frac{1}{2}$. Then, for any tax policy in $\mathcal{A}$, one can find an alternative one under which one type is made worse off and the other types are made better off. The latter is therefore preferred by a majority of individuals. It is exactly for this reason that a Downsian political economy approach to problem of non-linear income taxation has proven to be difficult.

Proposition B.4 is akin to Theorem 2. However, by assuming quasi-linear preferences we could prove a much stronger statement, namely that there is one specific policy that defeats any welfare-maximizing policy. Without the assumption of quasi-linearity, we can only show that to any welfare-maximizing policy there is an alternative policy that defeats it.

B.4 More political parties

Our main results extend to a model with more than two parties. We sketch the argument for the case of three parties, indexed by $j \in \{1, 2, 3\}$. We use the model of income taxation for illustration.

We study the best response problem of party 1. Given a policy $p^2 = (y^2, G^2)$ of party 2 and a policy $p^3 = (y^3, G^3)$ of party 3, it chooses $p^1 = (y^1, G^1)$ so as to maximize

$$E\left[\int_{\mathbb{R}_+} G^2(x_i^1 - h(\omega, y^1) + h(\omega, y^2))G^3(x_i^1 - h(\omega, y^1) + h(\omega, y^3))\, dG^1(x_i^1)\right] ,$$

subject to the constraint that y^1 is a non-decreasing function and subject to the budget constraint $\int_{\mathbb{R}_+} x^1\, dG^1(x_i^1) \leq e + S_v(y^1)$. Recall that $h(\omega, y^j)$ is the information rent a type ω-voter realizes given the function y^j, see the formal definition in equation (17). In a symmetric equilibrium, all parties propose the same income tax schedule so that, for all ω, $h(\omega, y_{eq}) =: h(\omega, y^1) = h(\omega, y^2) = h(\omega, y^3)$. In addition, $G_{eq} := G^1 = G^2 = G^3$. In order to characterize G_{eq}, we consider the income tax schedules as predetermined. The best response problem of party 1 then is to choose a distribution of pork G^1 so as to maximize

$$\int_{\mathbb{R}_+} G^2(x_i^1)\, G^3(x_i^1)\, dG^1(x_i^1),$$

subject to $\int_{\mathbb{R}_+} x^1\, dG^1(x_i^1) \leq e + S_v(y^1)$. Parties 2 and 3 face the same problem.

Again, this game has been analyzed by Myerson (1993). He has shown that, in a symmetric equilibrium,

$$G_{eq}(x) = \left(\frac{x}{3\,(e + S_v(y_{eq}))}\right)^{\frac{1}{2}} ,$$

for all $x \in [0, 3\,(e + S_v(y_{eq}))]$.[48]

To characterize y_{eq} we look once more at party 1's best response problem and exploit that $G^2 = G^3 = G_{eq}$ and that $y^2 = y^3 = y_{eq}$. Hence, the best response problem of party 1 is to choose G^1 and y^1 so as to maximize

$$E\left[\int_{\mathbb{R}_+} \left(G_{eq}(x_i^1 - h(\omega, y^1) + h(\omega, y_{eq}))\right)^2 \, dG^1(x_i^1)\right]$$
$$= E\left[\int_{\mathbb{R}_+} \frac{x_i^1 - h(\omega, y^1) + h(\omega, y_{eq})}{3\,(e + S_v(y_{eq}))} \, dG^1(x_i^1)\right]$$

subject to the budget constraint $\int_{\mathbb{R}_+} x_i^1 \; dG^1(x_i^1) \leq e + S_v(y^1)$. This objective is linear in x_i^1. This implies that, from here on, all the arguments in the proofs of Lemmas A.4, A.5 and Theorem 2 go through. Consequently, $y_{eq} = y^*$ and any deviation from y^* will imply a loss of votes relative to the equilibrium vote share.

[48]More generally, in the symmetric equilibrium of a model with m vote-share maximizing parties pork-barrel spending is determined by a distribution G_{eq} with $G_{eq}(x) = \left(\frac{x}{m(e+S_v(y_{eq}))}\right)^{\frac{1}{m-1}}$, for all $x \in [0, m\,(e + S_v(y_{eq}))]$.